Sensitivity of the Latitude of the Westerly Jet Stream to Climate Forcing

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Abstract

The latitude of the westerly jet stream is influenced by a variety of climate forcings, but their effects on the jet latitude often manifest as a tug of war between tropical forcing (e.g., tropical upper-tropospheric warming) and polar forcing (e.g., Antarctic stratospheric cooling or Arctic amplification). Here we present a unified forcing-feedback framework relating different climate forcings to their forced jet changes, in which the interactions between the westerly jet and synoptic eddies are synthesized by a zonal advection feedback, analogous to the feedback framework for assessing climate sensitivity. This framework is supported by a prototype feedback analysis in the atmospheric dynamical core of a climate model with diverse thermal and mechanical forcings. Our analysis indicates that the latitude of a westerly jet is most sensitive to the climate change-induced jet speed changes near the tropopause. The equatorward jet shift also displays a larger deviation from linearity than the poleward counterpart.

Plain Language Summary

The westerly jet stream is perceived as the bellwether of midlatitude storminess, surface temperature extremes, and precipitation. The effects of different climate forcings, such as greenhouse gas warming and Antarctic ozone hole, on the westerly jet often result in large cancelations. Here we show that the competing effects can be quantified by a unified forcing-feedback framework. Our analysis demonstrates the critical role in jet shift of the zonal wind changes near the tropopause, which are associated with changes in the equator-to-pole temperature gradient under climate change. We also found that the equatorward jet shift is larger than the linear feedback prediction, but this nonlinear effect is small for a poleward jet shift.

1. Introduction

The westerly jet streams are the prevailing midlatitude winds in Earth’s atmosphere, influencing regional storminess, surface temperature extremes, and precipitation (Shaw et al., 2016). Understanding what determines the latitudinal shift of midlatitude jet streams under climate change is a basic question in climate dynamics. Observations have witnessed a pronounced poleward shift in the Southern Hemisphere (SH) westerly jet over the past several decades (Figures 1b and 1d), which is largely attributed to greenhouse gas increases and Antarctic ozone depletion (Chen & Held, 2007; Lee & Feldstein, 2013; Son et al., 2008; Thompson et al., 2002). As greenhouse gases continue to rise, they are expected to cause more poleward shifts in midlatitude westerlies in both hemispheres. The future shifts in westerlies with greenhouse warming, however, are likely in a tug of war with the recovery of Antarctic ozone hole after 2000s in the SH (Eyring et al., 2013; Gerber & Son, 2014) and with the Arctic amplification in the Northern Hemisphere (NH) (Blackport & Kushner, 2017; Harvey et al., 2014; Peings et al., 2017). Other climate forcings such as aerosols or internal climate variability may have also contributed to the observed trends (Allen et al., 2014; Ming et al., 2011).

The complex two-way interactions between synoptic storms and jet streams have obscured a simple explanation of the jet stream response to climate change (Shaw, 2019). The variability of midlatitude westerly jet is often characterized by a latitudinal shift in zonal wind, with a coherent structure from the surface to the lower stratosphere (see the SH in Figures 1b and 1d). The structures resemble the leading annular modes of the extratropics, representing the unforced fluctuations in midlatitude jet due to synoptic eddy feedbacks (Chen & Plumb, 2009; Lorenz & Hartmann, 2001; Simpson et al., 2013). Theoretical studies on the applicability of the fluctuation-dissipation theorem showed that the jet response to idealized thermal
and mechanical forcings increases with greater annular mode feedback (Ring & Plumb, 2008). Analysis of global climate models (Barnes & Hartmann, 2010; Kidston & Gerber, 2010; Son et al., 2010) found that a westerly jet with stronger annular mode feedback is correlated with a larger latitudinal shift in response to climate forcing, but this relationship failed to explain the seasonality of jet shift (Simpson & Polvani, 2016). In this paper, we present a new forcing-feedback framework to quantify the sensitivity of the westerly jet to climate forcing, in contrast to the traditional approach focusing on the eddy-mean flow interactions in the jet response to climate change (see the review by Shaw, 2019).

The following sections are organized as follows. Section 2 describes the forcing-feedback framework for the jet response to climate forcing. In section 3, we present a prototype of feedback analysis in the atmospheric dynamical core, where the zonal advection feedback is simulated in a global climate model. Our feedback analysis identifies that the latitudinal shift in the westerly jet is most sensitive to climate change-induced jet speed changes near the tropopause. Conclusions and discussion are provided in section 4. Details of model configurations and feedback analysis are summarized in the appendixes.

2. Forcing-Feedback Framework for the Jet Sensitivity

The response of a climate variable to climate forcing is generally quantified in the forcing-feedback framework of a dynamical system (Roe, 2009). This feedback framework is well established for global surface warming under greenhouse gas increases: we first compute the surface warming in response to the radiative forcing of rising greenhouse gases in a reference climate system without any feedback and then calculate the additional radiative forcing (i.e., feedbacks) resulting from the prior surface warming, which, in turn, cause further surface warming and radiative forcing until the climate system reaches a new radiative equilibrium.

This climate sensitivity framework only evaluates globally averaged variables, but understanding circulation shift must consider the spatial variations in atmospheric circulation. We consider the spatial structure of zonally averaged zonal wind, denoted in a vector form as $\mathbf{Z}$, with the dimension of $N \times 1$ ($N$ is the number of latitudinal grid points times the number of vertical levels). Similar to climate sensitivity, we evaluate the zonal wind response to climate forcing in two steps: (i) the direct zonal wind response to the effective zonal

![Figure 1](image-url). Linear trends of temperature and zonal wind in observations and global climate models. December–February (DJF) and zonally averaged (left) temperature (K decade$^{-1}$) and (right) zonal wind (m s$^{-1}$ decade$^{-1}$) for (a, b) observations (ERAI reanalysis) and (c, d) CMIP5 multimodel means in 1979–2016. For CMIP5 models, the historical outputs during 1979–2005 and RCP8.5 outputs during 2006–2016 are used here. The black contours in individual panels indicate their climatological means. See the data source in Appendix A.
momentum forcing under climate change and (ii) additional feedbacks resulting from the changes in zonal wind. More specifically, the governing equation for $Z$ is written as (Ring & Plumb, 2008)

$$\frac{\partial Z}{\partial t} + MZ = E \tag{1}$$

where $M$ is an $N \times N$ matrix describing all the processes for the zonally symmetric dynamics, including the horizontal and vertical advection and thermal and frictional damping rates. $E$ is an $N \times 1$ vector representing all the eddy terms that are not resolved in the zonally symmetric dynamics.

Let us apply climate forcing that induces a sustained perturbation in either angular momentum or diabatic heating to the atmosphere so as to produce an effective zonal momentum forcing $F$. In a new equilibrium state the changes in zonal mean zonal wind are

$$\Delta Z = M^{-1}(F + \Delta E). \tag{2}$$

where $\Delta E$ is the change in the eddy terms.

For a small climate perturbation, the changes in eddies can be expanded with respect to the changes in zonal wind as

$$\Delta E = \Delta E_0 + \frac{\partial E}{\partial Z} \Delta Z \tag{3}$$

where $\partial E / \partial Z$ is an $N \times N$ matrix, and the higher-order terms are neglected. $\Delta E_0$ is the eddy changes independent of the changes in zonal flow (to be explained later). $\partial E / \partial Z$ quantifies the strength of eddy feedback due to the changes in zonal flow. Following previous studies (Chen & Plumb, 2009; Lorenz & Hartmann, 2001; Ring & Plumb, 2008), we have restricted the eddy feedback to that associated with the variation in zonal wind, and one may choose to further analyze the barotropic and baroclinic processes associated with the variability in zonal wind (Burrows et al., 2017; Nie et al., 2014). Our choice also stems from the Taylor expansion with respect to our target variable of zonal wind.

In the absence of the eddy feedback, the direct response in zonal mean zonal wind is obtained as

$$\Delta Z_0 = M^{-1}(F + \Delta E_0) \tag{4}$$

Here $\Delta E_0$ accounts only for the eddy changes independent of the zonal wind changes (e.g., the eddy response to a change in static stability). Including the eddy feedback, the full response in zonal mean zonal wind is

$$\Delta Z = M^{-1}(F + \Delta E_0 + \frac{\partial E}{\partial Z} \Delta Z) \tag{5}$$

Now defining a circulation feedback matrix $F$ as

$$F = M^{-1} \frac{\partial E}{\partial Z}, \tag{6}$$

collecting all the terms with $\Delta Z$ yields

$$\Delta Z = (I - F)^{-1} \Delta Z_0 \tag{7}$$

Here $I$ is the identity matrix. $(I - F)^{-1}$ is the factor that the zonal wind has gained from the feedback, with $F$ as a generic measure of the eddy feedback. This quantifies the relation that greater positive feedback leads to larger amplification in zonal wind. As such, the full zonal wind response to climate forcing $\Delta Z$ can be understood, via equation (7), as the direct response $\Delta Z_0$ together with the nondimensional feedback matrix $F$ or the gain matrix given by $(I - F)^{-1}$.

While the above eddy feedback is generic, we will only analyze the zonal advection feedback that can be implemented in a climate model by a two-step procedure (Figure S1): (i) the direct response induced by climate forcing when the zonal mean zonal wind for the advection of vorticity and temperature is the same as the reference climate and (ii) the zonal advection feedback from the changes to the zonal mean zonal wind. This is motivated not only to ensure that the model implementation does not violate the global energy conservation (see the overriding method in Appendix C) but also to understand the effect of zonal wind changes on eddy phase speed as a mechanism for the jet response to climate change (Chen & Held, 2007;
As such, this zonal advection feedback is stricter than the generic eddy feedbacks considered in the literature (e.g., the eddy feedback from changes in static stability may be excluded). To compute the zonal advection feedback matrix $F$, we need to first construct an overriding version of the atmospheric dynamical core, in which the zonal mean zonal wind for zonal advection is prescribed so as to disable the zonal advection feedback, analogous to prescribing the SST to an atmospheric model to disable the radiative feedback associated with surface warming. Then, the feedback matrix is estimated by systematically perturbing the prescribed zonal mean zonal wind, analogous to the SST perturbations for Cess climate sensitivity (Cess et al., 1990). The comparison between the jet sensitivity and Cess climate sensitivity are itemized in Table S1. The spatial structure of the feedback matrix is obtained through the Green's function method described in Appendix D.

Other variables (e.g., temperature), denoted as $X$, can vary consistently with zonal mean zonal wind. Similar to equation (3), a Taylor expansion of $X$ with the changes in zonal wind can be obtained. Substituting with equation (7), $\Delta X$ can be predicted by the direct responses in $X$ and $Z$ as

$$\Delta X = \Delta X_0 + F_X(I - F)^{-1}\Delta Z_0$$  (8)

where $\Delta X_0$ is the direct response independent of the zonal advection feedback, and $F_X = \frac{\partial E}{\partial Z}$ is the corresponding feedback matrix for $X$. The feedback matrix $F_X$ is calculated using the same method as $F$ and is described in Appendix D.

3. Results

3.1. The Importance of Feedback for Time Mean Response

Changes to atmospheric circulation can be ultimately traced back to the radiative effect of climate forcing: Antarctic ozone hole produces radiative cooling in the polar lower stratosphere (Thompson et al., 2002); increases in greenhouse gases lead to enhanced tropical upper-tropospheric warming and Arctic amplification by the interactions between atmospheric radiation and dynamics (Blackport & Kushner, 2017; Peings et al., 2017). These thermal fingerprints are evident in both observations and global climate models (Figures 1a and 1c), which can be used to infer the forced circulation signals with respect to competing climate forcings (Blackport & Kushner, 2017; Gerber & Son, 2014; Lee & Feldstein, 2013; Peings et al., 2017; Son et al., 2008).

We develop a prototype of feedback analysis for the zonal advection feedback using the atmospheric dynamical core of a global climate model. The dynamical core can be run in a simple modeling framework that is forced by Newtonian relaxation of temperature and dissipated by Rayleigh damping in the planetary boundary layer. While this simple framework can produce reasonably realistic global atmospheric circulation, for the purpose of reproducibility we use the standard benchmark calculation for the dynamical core (Held & Suarez, 1994) (see Appendix B for details). Moreover, we force the dynamical core with climate change-like thermal perturbations, modified from those used in Butler et al. (2010), including Antarctic stratospheric cooling, tropical upper-tropospheric heating, and Arctic near-surface heating (Figure S2). As such, this modeling framework retains the fluid dynamics of global atmospheric circulation but simplifies the physical processes to thermal forcing and frictional dissipation, permitting a comprehensive examination on the sensitivity of the westerly jet to a variety of thermal and mechanical forcings. We note that the control climate and tropical heating are hemispherically symmetric and that the cross-hemispheric responses to polar forcings are negligible, and thus only hemispherically symmetric Green's function perturbations and feedback modes are considered here to reduce the computational cost.

The dynamical core can reasonably capture the structure of zonal wind changes in observations and global climate models (cf. Figure 1 and Figures 2a and 2b). As in Butler et al. (2010), these simulations show that tropical upper-tropospheric warming works constructively with Antarctic stratospheric cooling in shifting the SH westerly jet poleward, whereas it has the opposite effect to Arctic near-surface warming on the jet latitude in the NH (Figures 2a, 2b, S3, and S4). Nevertheless, while the pattern of temperature response is similar to the prescribed thermal forcing (cf. Figures 2a and S2), the Antarctic stratospheric cooling is much weaker when the zonal advection feedback is disabled (cf. Figures 2c and 2a). The strengthening of the SH westerly jet is consistent with polar stratospheric cooling, but the direct zonal wind response without feedback displays almost no latitudinal shift in the SH. In contrast, the predicted temperature and zonal wind from equation (7), albeit slightly weaker in magnitude, agree with the full responses reasonably well.
Figure 2. Response to climate change-like thermal forcing in the atmospheric dynamical core. Time and zonally averaged (left) temperature (K) and (right) zonal wind (m s$^{-1}$), for (a, b) the full responses, (c, d) the direct responses when the zonal advection feedback is disabled, and (e, f) the predictions from the forcing-feedback framework using equation (7). The black contours in individual panels indicate their climatological means. The thermal forcing used is displayed in Figure S2. The responses of temperature and zonal wind to individual regions of the thermal forcing are shown in Figures S3 and S4.

This suggests that the westerly jet shift is sustained by the zonal advection feedback. As a large portion of Antarctic stratospheric cooling in the dynamical core is the effect of, rather than causing, the circulation shift, the statistical correlation between polar stratospheric cooling and the latitudinal shift of tropospheric jet found in global climate models (Eyring et al., 2013; Gerber & Son, 2014; Son et al., 2008) should not be interpreted as a simple one-way influence from the ozone hole on the tropospheric jet but as a balanced response of two-way interactions.

3.2. Leading Mode of Zonal Advection Feedback

The zonal advection feedback can be understood by a modal decomposition of the feedback matrix. We apply the singular vector decomposition (SVD) analysis, yielding $(I - F)^{-1} = U \Sigma U_0^T$, where $U$ and $U_0$ are the left and right singular vectors of the gain matrix, $\Sigma$ is a diagonal matrix with singular values, and the superscript $T$ denotes transpose. Thus, equations (7) and (8) become

$$\Delta Z = U \Sigma U_0^T \Delta Z_0$$

$$\Delta X = \Delta X_0 + F_X \Sigma U_0^T \Delta Z_0$$
Figure 3. The leading feedback mode and associated temperature and circulation in the atmospheric dynamical core. (a) Left singular vector of the dimensionless feedback matrix, $U^1$; (b) right singular vector, $U^0_1$; (c) temperature (K); and (d) mean meridional circulation (10$^9$ kg m$^{-2}$). All values are rescaled relative to one standard deviation of $U^1$. The black contours in individual panels indicate their climatological means. Because the control climate and tropical heating are hemispherically symmetric and the cross-hemispheric responses to polar forcings are negligible, only the hemispherical symmetric feedback modes are examined here, and only one hemisphere is shown.

Projecting the zonal mean zonal wind onto the $k$th left and right singular vectors as $\Delta z_k^x = (U_k^x)^T \Delta Z$ and $\Delta z_0^{x_0} = (U_0^x)^T \Delta Z_0$, equation (9) gives $(\Delta z_k^x)^2 = \sigma_k^2 (\Delta z_0^{x_0})^2$, where $\sigma_k$ is the $k$th singular value of $\Sigma$.

If the direct forced zonal wind response does not project preferentially to the right vectors (i.e., evenly distributed $(\Delta z_k^{x_0})^2$), the variance of full zonal wind response $(\Delta z_k^x)^2$ will be dominated by the leading few singular vectors with the largest singular values, which we refer to as the leading modes of zonal advection feedback. As a result, the first left singular vector $U^1$ describes the leading zonal wind pattern of the full response to arbitrary climate forcing. The first right singular vector $U^0_1$ represents the leading zonal wind pattern of the direct response, alluding to the preferred climate forcing structure via equation (4). $F_X U^1$ is the leading pattern of $X$ (e.g., temperature) associated with the zonal advection feedback.

The left singular vector of the leading feedback mode (Figure 3a) indicates a meridional shift in tropospheric jet, which resembles the zonal wind pattern of the full response to climate change-like thermal forcing (Figure 2b). In contrast, the right singular vector of the leading mode (Figure 3b) exhibits largest signals above the westerly jet near the tropopause, indicating that the most effective way for a poleward jet shift is the zonal wind acceleration in the poleward and upward regions of the mean jet. This is supported by the comparison between the direct and full responses (Figures 2d and 2f). Note both singular vectors are rescaled relative to one standard deviation of $U^1$, and $\Delta z^1 = \sigma_1 \Delta z_0^1$ indicates amplification of the modal projection in zonal wind by the singular value of $\sigma_1 = 5.1$.

The leading feedback mode is accompanied by changes in temperature and mean meridional circulation. A poleward-shifted jet is associated with cooling in the lower-stratospheric high latitudes and warming in the lower-stratospheric low latitudes and tropospheric midlatitudes (Figure 3c), implying a jet-induced change in tropopause height. This corroborates that the circulation feedback from the poleward shift contributes...
Figure 4. Robust circulation feedback to diverse climate forcings. Predicted changes in (a) jet latitude and (b) Hadley cell edge are plotted against those from the full responses. The climate change-like thermal forcing is divided into Antarctic, tropic, and Arctic forcings, and individual forcing is varied gradually with both positive and negative signs; alternative forcings include changes to equator-to-pole temperature gradient, stratospheric temperature, and surface friction (Table S2). The westerly jet latitude is evaluated at 875 hPa. The Hadley cell edge is measured by the zero-crossing latitude of the 500-hPa mean meridional circulation. Positive (negative) values of the jet latitude or Hadley cell edge depict poleward (equatorward) movement. The responses to the Arctic (Antarctic) forcing are shown only for the Northern (Southern) Hemisphere, and the response to the hemispherically symmetric forcings is shown as the average of the two hemispheres.

largely to Antarctic stratospheric cooling in the full response to climate change-like forcing (Figure 2e). Changes to the westerly jet can also lead to changes to the Hadley cell and Ferrel cell, with a poleward movement of the westerly jet accompanying a poleward expansion of the Hadley cell and a poleward shift of the Ferrel cell (Figure 3d).

3.3. Robust Circulation Feedback to Diverse Forcings

The forcing-feedback framework assumes the feedback represents the internal dynamics of the atmosphere, independent of the types of climate forcing used (i.e., equation (3)). We test this assumption by forcing the atmospheric dynamical core with a number of thermal and mechanical perturbations. To this end, the climate change-like thermal forcing is divided into Antarctic, tropic, and Arctic regions, and the magnitude of individual thermal perturbation in each region is varied gradually with both positive and negative signs. Alternative forcings are examined, with changes to equator-to-pole temperature gradient (Gerber & Vallis, 2007), stratospheric temperature (Lorenz & DeWeaver, 2007), and surface friction (Chen et al., 2007). Details of these forcings are described in the supporting information. Examples of the changes in temperature and zonal wind under these forcings and their predicted changes are displayed in Figures S5–S7.

Despite the variety of how the forcing is prescribed, a relatively compact relation is found between the predicted changes to the westerly jet latitude and Hadley cell boundary and their full response counterparts (Figure 4), but with a nonlinear behavior deviating from the linear feedback prediction. The relative compactness supports the fact that both the linear feedback and the nonlinear deviation from linearity are intrinsic to the dynamical system. While the predicted poleward movements are slightly below the 1:1 line, the slope of predicted equatorward shifts are much smaller than the 1:1 line. Fitting the data with an extra second-order polynomial agrees well with the model results, and the linear component still accounts for about 60% of the full response. As the equatorward jet shift exhibits increased nonlinearity, this nonlinearity may be related to the interaction between the Hadley cell and midlatitude westerly jet through the merging of the equatorward-shifted eddy-driven jet with a subtropical jet (Lee & Kim, 2003). This indicates that additional higher-order terms should be considered to make more accurate predictions of circulation shift. Note despite the underprediction in magnitude, the spatial patterns of zonal wind changes are well predicted (Figures S4 and S5). Because the jet responses to different forcings are often assumed to be linearly additive.
this nonlinear effect suggests that caution should be taken in interpreting the jet response to competing climate forcings.

4. Conclusions and Discussion
We have developed a new forcing-feedback framework for assessing the sensitivity of the westerly jet to climate forcing in the same spirit as that of climate sensitivity. This framework divides the westerly jet response to climate forcing into two components: (i) the direct response induced by climate forcing when the zonal mean zonal wind for the advection of vorticity and temperature is the same as the reference climate and (ii) the zonal advection feedback from the changes to the zonal mean zonal wind. We note this feedback is stricter than the generic eddy feedbacks discussed in the literature (e.g., the eddy feedback associated with changes in static stability may be excluded). Because it is directly evaluated in an atmospheric model as for climate sensitivity, it enables a quantitative assessment of the feedback processes in the jet response to climate change.

We provide a prototype of feedback analysis for the jet response to a variety of thermal and mechanical forcings in an atmospheric dynamical core, demonstrating that the advection feedback contributes largely to the mean jet response to climate forcing. The leading mode of zonal advection feedback indicates that the westerly jet latitude is most sensitive to the zonal acceleration near the tropopause, with associated changes in temperature, tropopause height, the Hadley cell, and Ferrel cell (Figure 3). This helps explain the sensitivity of tropospheric jet to tropical upper-tropospheric warming versus Arctic near-surface warming found in comprehensive climate models (Blackport & Kushner, 2017; Peings et al., 2017). That the tropospheric jet shift can enhance the polar stratospheric cooling implies an important feedback onto the stratospheric cooling, as compared with the simple one-way influence from stratospheric ozone on the tropospheric jet (Eyring et al., 2013; Gerber & Son, 2014; Son et al., 2008). Moreover, we found an increased deviation from linearity for an equatorward-shifted jet in comparison with the poleward counterpart (Figure 4). This is consistent with the nonlinear interactions between the Hadley cell and westerly jet through the merging of the equatorward-shifted eddy-driven jet with the subtropical jet (Lee & Kim, 2003).

Appendix A: Observations and Climate Models
The observed trends of the zonal wind and temperature are calculated from the European Centre for Medium-Range Weather Forecasts (ECMWF) Interim Re-Analysis (ERA-Interim) (Dee et al., 2011).

The 27 CMIP5 climate models used are ACCESS2.0, ACCESS2.3, BCC-CSM1.1, BCC-CSM1.1.m, CanESM2, CCSM4, CESM1-CAM5, CSIRO-Mk3.6.0, FGOALS-g2, GFDL-CM3, GFDL-ESM2G, GFDL-ESM2M, GISS-E2-H, GISS-E2-H-CC, GISS-E2-R, HadGEM2-CC, HadGEM2-ES, INMCM4, IPSL-CM5A-MR, IPSL-CM5B-LR, MIROC5, MIROC-ESM, MIROC-ESM-CHEM, MPI-ESM-LR, MPI-ESM-MR, NorESM1-M, and NorESM1-ME.

Appendix B: Atmospheric Dynamical Core
We use the Geophysical Fluid Dynamical Laboratory (GFDL) spectral atmospheric dynamical core (Held & Suarez, 1994). The model solves the primitive equations on the sphere with the pseudospectral method. Temperature is relaxed to a prescribed radiative equilibrium temperature profile. Surface drag is parameterized as Rayleigh friction in the planetary boundary layer. The subgrid-scale diffusion is parameterized by the $\Delta^8$ hyperdiffusion on temperature, vorticity, and divergence. Readers are referred to Held and Suarez (1994) for more details.

All of the model simulations are conducted at T42 horizontal resolution (whose Gaussian grid is roughly $3^\circ \times 3^\circ$) with 20 equally spaced sigma levels in the vertical ($\sigma = p/p_s$ and $p$ is pressure and $p_s$ is surface pressure). This gives the dimension of zonal mean zonal wind as $N = 64 \times 20 = 1280$. The simulations are integrated for 18 years, with the first 3 years discarded as the spinup.

Appendix C: Overriding Method
The zonal advection feedback matrix is computed using an overriding version of the dynamical core, in which the zonal mean zonal wind felt by eddies is specified as the zonal wind taken from the standard
control simulation, but the zonal mean zonal wind remains interactive in the overriding model. This is implemented in the model by overriding the effects of zonal mean zonal wind on the zonal advection of eddies. More specifically, the advection of vorticity, $\zeta$, and temperature, $T$, is modified as follows

$$\frac{D\delta\zeta}{Dt} = \frac{D\zeta}{Dt} + \frac{(\vec{u}_{\text{ovrd}} - \vec{u})}{a \cos \phi} \frac{\partial \zeta}{\partial \lambda}$$

$$\frac{D\delta T}{Dt} = \frac{DT}{Dt} + \frac{(\vec{u}_{\text{ovrd}} - \vec{u})}{a \cos \phi} \frac{\partial T}{\partial \lambda}$$  \hspace{1cm} (C1)

where $\lambda$ is longitude, $\phi$ is latitude, and $a$ is Earth’s radius. $D/Dt$ represents the total derivative of vorticity or temperature. Overbars denote zonal means. As such, the zonal mean zonal wind in the zonal advection of eddies $\vec{u}$ is replaced by $\vec{u}_{\text{ovrd}}$, and the model-generated zonal mean zonal wind cannot influence the eddies directly by zonal advection. In practice, $\vec{u}_{\text{ovrd}}$ uses the 6-hourly zonal mean zonal wind from the standard control run. The 6-hourly data are linearly interpolated in time within the 6-hourly interval to provide a continuously varying time series as input for the overriding model.

The overriding formulation does not directly modify the zonal mean of the $n$th power of vorticity and temperature (i.e., $\frac{D\delta\zeta^n}{Dt} = \frac{D\zeta^n}{Dt} + \frac{(\vec{u}_{\text{ovrd}} - \vec{u})}{a \cos \phi} \frac{\partial \zeta^n}{\partial \lambda}$). This ensures the global energy conservation in the overriding model. The overriding method can be thought of as modifying the zonal propagation speed of eddies by $\vec{u}_{\text{ovrd}} - \vec{u}$, similar to a Doppler shift. This can be regarded as a test for the modification of zonal wind on eddy phase speed for the jet response to climate change (Chen & Held, 2007; Chen et al., 2008).

Appendix D: Green’s Functions

The zonal advection feedback matrix $F$ is calculated by Green’s functions with $N$ different perturbations of $\delta Z_{\text{ovrd}}$ (i.e., the vector form of $\vec{u}_{\text{ovrd}}$) via matrix inversion, similar to Hassanzadeh and Kuang (2016).

$$F = \delta Z (\delta Z_{\text{ovrd}})^{-1}$$  \hspace{1cm} (D1)

Here $F$ is an $N \times N$ matrix. $\delta Z_{\text{ovrd}}$ and $\delta Z$ are $N \times 1$ matrices, and the matrix inversion can be performed with $N$ independent perturbations of $\delta Z_{\text{ovrd}}$. Here $\delta$ has the same meaning as $\Delta$ used in other sections, but it is used to distinguish the Green’s function analysis from the response to climate forcing. Similarly, the feedback matrix for $X$ can be obtained from the Green’s function analysis as

$$F_X = \delta X (\delta Z_{\text{ovrd}})^{-1}$$  \hspace{1cm} (D2)

To decrease the computational cost of the Green’s function analysis, we reduce the dimension of Green’s function perturbations by choosing the 110 basis functions as in Hassanzadeh and Kuang (2016).

$$b_{(\phi_{j}+10k)} = \exp[-(\frac{|\phi - \phi_{j}|}{10^o})^2 - (\frac{\sigma - \sigma_{k}}{0.075})^2]$$  \hspace{1cm} (D3)

where $j = 1, 2, \ldots , 10$, and $k = 0, 1, \ldots , 10$. The center latitudes of the basis functions are $\phi_{j} = 0^o, 10^o, \ldots , 90^o$, and the center sigma levels are $\sigma_{k} = 0, 0.1, \ldots , 1$. For efficiency, we have also taken advantage of the hemispheric symmetry in tropical upper-tropospheric warming, and the cross-hemispheric responses to thermal perturbations in the polar regions are negligible, and thus only hemispheric symmetric Green’s function perturbations are considered here.

All the off-line feedback analysis are based on the projections of zonal mean zonal wind onto these basis functions through the least squares regressions: $\delta Z = b\delta Z_{\text{ovrd}}$, and $\delta Z_{\text{ovrd}} = b\delta Z_{\text{ovrd}}$, where the dimension of $b$ is $N \times 110$, and the tildes represent the projections with the dimension of $110 \times 1$ as compared with the original fields with the dimension of $N \times 1$. The feedback analysis uses these projections rather than the original zonal wind fields, which is justified by the large scale features of the zonal mean zonal wind. This projection reduces the required number of independent perturbations for matrix inversion from $N = 1, 280$ at current resolution to 110.
The Green function analysis uses 110 perturbations of $\delta Z^{\text{ord}}$ with a magnitude of 12 m s$^{-1}$ and of both positive and negative signs, yielding a diagonal perturbation matrix as

$$
\delta Z^{\text{ord}}_{10x110} = 12 \text{ m s}^{-1}
$$

The difference of positive and negative perturbations can eliminate the errors in estimating the linear response $\delta \bar{Z} = (\delta Z^+ - \delta Z^-)/2$, where the plus sign corresponds to positive perturbations and minus to negative perturbations, respectively. The response in $\delta \bar{Z}$ divided by the perturbation matrix $\delta Z^{\text{ord}}$ yields a feedback matrix as

$$
\bar{F} = \delta \bar{Z}_{10x110}(\delta Z^{\text{ord}}_{10x110})^{-1}
$$

Because the matrix $\bar{F}$ is well-conditioned, we do not need to perform any filtering prior to matrix inversion, as compared with Hassanzadeh and Kuang (2016).

We do not project any other fields, $X$, onto these basis functions, as they may have small-scale structures that are not well represented by these basis functions. The feedback matrix for $X$ is obtained as

$$
\bar{F}_X = \delta X_{Xx110}(\delta Z^{\text{ord}}_{10x110})^{-1}
$$

References


