Sensitivities and mechanisms of the zonal mean atmospheric circulation response to tropical warming

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Although El Niño and global warming are both characterized by warming in the tropical upper troposphere, the latitudinal changes of the Hadley cell edge and mid-latitude eddy-driven jet are opposite in sign. Using an idealized dry atmospheric model, the zonal mean circulation changes are investigated with respect to different patterns of tropical warming. Generally speaking, an equatorward shift in circulation takes place under strong tropical temperature gradient or narrow tropical warming, similar to the changes associated with El Niño events. In contrast, the zonal mean atmospheric circulations expand or shift poleward in response to upper tropospheric warming or broad tropical warming, resembling the changes under future global warming.

The mechanisms of the opposite changes in circulation are investigated by comparing between the dry dynamical responses to a narrow tropical warming and a broad warming as analogues for El Niño and global warming. When running the idealized model in a zonally symmetric configuration in which the eddy feedback is disabled, both the narrow and broad warmings give rise to an equatorward shift of the subtropical jet. The eddy adjustment is further examined using large ensembles of transient response to a sudden switch-on of the forcing. For both narrow and broad tropical warmings, the jets move equatorward initially. In the subsequent adjustment, the initial equatorward shift is further enhanced and sustained by the low level baroclinicity under the narrow tropical warming, whereas the initial equatorward shift transitions to a poleward shift associated with altered irreversible mixing of potential vorticity in the upper troposphere under the broad warming.
1. Introduction

It is well known that tropical heating can impact global atmospheric circulation and climate. During the warm phase of the El Niño-Southern Oscillation (ENSO), the enhanced tropical convection over the equatorial Eastern Pacific, can not only excite a zonally asymmetric Rossby wave teleconnection pattern (e.g., Hoskins and Karoly 1981), but also alter the tropical Hadley cell circulations and midlatitude zonal jets (Robinson 2002; Seager et al. 2003; L’Heureux and Thompson 2006; Lu et al. 2008; Chen et al. 2008; Gong et al. 2010). The mechanisms for the contraction of the Hadley cell and the equatorward displacement of midlatitude jet in response to the enhanced tropical heating during El Niño have been extensively studied and relatively well understood. Using an idealized atmospheric model, Chang (1995) investigated the effect of the Hadley circulation intensity on the extratropical winds by enhancing the meridional gradient of the tropical heating and found an equatorward shift of the jet stream associated with a stronger Hadley circulation, similar to the changes observed during El Niño. This extratropical response to the El Niño-like tropical heating can be understood as the impact of subtropical zonal wind anomalies on the equatorward propagation and absorption of the midlatitude eddies near their critical latitudes, and subsequently on the eddy-driven extratropical circulation (e.g., Chang 1998; Robinson 2002; Seager et al. 2003; Chen et al. 2008). Additionally, the anomalous subtropical winds can affect the type or frequency of nonlinear wave breaking in the subtropics and thus shift the eddy-driven wind (e.g., Orlanski 2003; Gong et al. 2010). These studies suggest if the thermally-forced subtropical zonal wind is enhanced, the extratropical eddy-driven wind would shift equatorward by the changes in either quasi-linear wave propagation or nonlinear
wave breaking.

The enhanced equatorial SST warming (Liu et al. 2005; Vecchi et al. 2008; Collins et al. 2010; Lu and Zhao 2012) under increased greenhouse gases (GHGs) is also expected to give rise to a stronger subtropical jet by a thermal wind relationship. As demonstrated in Lu et al. (2008) and Chen et al. (2008), both El Niño and GHG forcings can give rise to an enhancement of the subtropical wind on the jet’s equatorial flank. More recently, by forcing an aqua-planet atmospheric model with an abrupt switch-on of a 4-Kelvin uniform SST warming, the initial phase of the ensemble adjustment is characterized by an intensification of the Hadley cell and subtropical zonal wind (Chen et al. 2012), a pattern resembling that forced by the enhanced gradient of the tropical heating in Chang (1995). The similar subtropical wind response between El Niño and global warming is rooted in the fundamental Hadley circulation dynamics: the small Coriolis parameter near the equator and the nearly angular-momentum conserving tropical wind cannot sustain a large scale temperature gradient and the effect of the tropical heating can spread over the entire tropics, thus fixating the anomalous temperature gradient and associated thermal wind near the edge of the tropics (e.g., Held and Hou 1980; Bretherton and Sobel 2003). If the same eddy adjustment as observed during El Niño were operating in isolation, the enhanced equatorial SST warming and associated subtropical wind under climate warming should have given rise to a similar equatorward shift in circulation. Taken at face value, it seems paradoxical that under climate warming scenarios, the Coupled Model Inter-comparison Project phase 3 (CMIP3) models tend to simulate a poleward expansion of the Hadley cell (Lu et al. 2007, 2008) and a poleward shift of the midlatitude storm tracks in both hemispheres (e.g., Yin 2005; Lorenz and DeWeaver 2007).
To further complicate the matter, in studying the sensitivities of the circulation change to the tropical heating in a dry atmospheric model, Butler et al. (2010) found no equatorward shift of the jet stream or contraction of the Hadley cell in response to tropical heating prescribed in the upper troposphere, even when the tropical heating is confined in the deep tropics. This seems to be contradictory to other idealized studies with dry (e.g., Chang 1995; Robinson 2002; Son and Lee 2005) or moist models (e.g., Chen et al. 2010). We suspect that these apparent discrepancies are the results of the sensitivity of the circulation response to both the horizontal and vertical structures of the tropical heating. Therefore, using a dry dynamical model similar to previous idealized studies, we first explore the sensitivities of circulation with respect to the tropical temperature gradient as in Chang (1995) and the width of the tropical upper tropospheric warming as in Butler et al. (2010). We find consistent results to the previous studies and the aforementioned seemingly controversial sensitivities can be reconciled within a single dynamical model. Further, we identify an intriguing set of experiments in which an equatorward-to-poleward transition in circulation occurs as the meridional extent of tropical heating is broadened. This set of experiments is believed to hold the key to understanding the distinct mechanisms between the equatorward “El Niño-like” response versus the “global warming-like” poleward response to tropical heating.

The challenge of understanding the effect of the tropical heating arises from the intricate interplay between the thermally forced component of the circulation and the eddy adjustment to it. To isolate the effect of the eddy adjustment, we make use of the zonally symmetric version of our model to achieve the thermally forced response without the effect of the eddy feedbacks. Through this approach, we demonstrate that the eddy feedbacks are indispensable in the equilibrium solution as the zonal mean circulation undergoes an
equatorward-to-poleward transition with gradually widened tropical warming. To further diagnose the eddy adjustment processes, a finite-amplitude wave activity budget is developed following Nakamura and Zhu (2010) and Nakamura and Solomon (2010). This hybrid Eulerian-Lagrangian formalism of the wave activity budget overcomes the difficulty of defining the wave activity in the regions of vanishing potential vorticity (PV) gradient. Since this diagnostic framework applies to both small-amplitude and finite-amplitude eddies, it aids in revealing the effect of the irreversible processes on the zonal jets, which are crucial for discerning the distinct mechanisms between the equatorward and poleward circulation shifts.

The paper is structured as follows. In section 2, we will briefly describe the dry atmospheric model, its zonally symmetric version, and the thermal forcings used for the perturbation experiments. The equilibrium responses to three sets of thermal perturbations will be discussed in Section 3. The results of the large-size ensemble transient experiments will be presented and discussed in Section 4. A novel diagnostic using the framework of finite-amplitude wave activity budget will be performed in section 5 to probe into the mechanisms of the eddy feedback during the transient response to tropical warming. The paper will conclude with discussion and conclusion.
2. Model description and experiment setup

a. Full model and thermal perturbations

We use the Geophysical Fluid Dynamics Laboratory (GFDL) atmospheric dynamical core with the Held and Suarez (1994) forcing. More specifically, the model is forced by a relaxation towards a prescribed zonally symmetric radiative equilibrium temperature profile and damped by a linear friction in the planetary boundary layer. In the control simulation, the radiative equilibrium temperature is specified as

$$T_{eq}^c(\phi, p) = \max\{200K, [315K - \delta_y \sin^2 \phi - \delta_z \log\left(\frac{p}{p_0}\right) \cos^2 \phi] \cdot \left(\frac{p}{p_0}\right)\}$$  \hspace{1cm} (1)

where $\delta_y=60K, \delta_z=10K$. All the $T_{eq}$ profiles used in this study are hemispherically symmetric.

All integrations are performed at the spectral resolution of R30 (rhomboidal 30 spherical harmonic truncations) with 20 evenly spaced sigma levels in the vertical.

In the perturbation experiments, diabatic heating is introduced by modifying the equilibrium temperature in the tropical troposphere. The change in the radiative equilibrium temperature multiplied by the thermal damping rate yields the perturbation in diabatic heating rate. Three sets of thermal perturbations are examined by adding different perturbations to the box in the equation 1, so that

$$T_{eq}^p(\phi, p) = \max\{200K, [315K - \delta_y \sin^2 \phi + F(\phi, p)W(\phi, \phi_0)] - \delta_z \log\left(\frac{p}{p_0}\right) \cos^2 \phi] \cdot \left(\frac{p}{p_0}\right)\}$$  \hspace{1cm} (2)

where $F(\phi, p)$ is the thermal perturbation, $W(\phi, \phi_0) = 0.5(1 - \tanh((|\phi| - \phi_0)/\delta))$ is a weighting function to specify the meridional structure of thermal perturbations, with $\phi_0$ setting the boundary of the thermal perturbation and $\delta\phi=5^\circ$ controlling the sharpness of the boundary. Within each set of experiment, we test the sensitivities of the equilibrium
solution to different widths of the heating by varying $\phi_0$ from $10^\circ$ to $30^\circ$ with an increment of $5^\circ$. The formulations are summarized as follows, and the parameters for the three sets of thermal perturbations are listed in table 1.

- **Tropical Temperature Gradient (TTG):** To mimic the configuration in Chang (1995), the meridional gradient of diabatic heating profile is enhanced in the deep tropics by using,

$$F(\phi, p) = A + \sin^{1.25}(|\phi|) - \sin^2(\phi)$$

The constant $A$ is included in the formulation to keep the global mean equilibrium temperature unchanged. As a result, this configuration leads to warming at the equator and cooling away from the equator.

- **Tropical Upper-tropospheric Warming (TUW):** Approximately following the heating profiles used in Butler et al. (2010), a tropical heating centered at the upper troposphere is added by using,

$$F(\phi, p) = A \cdot \exp(-\frac{(p/p_s - 0.3)^2}{2 \cdot 0.11^2})$$

where $A$ denotes the magnitude of the warming, $\exp(-\frac{(p/p_s - 0.3)^2}{2 \cdot 0.11^2})$ controls the vertical extents of the upper tropospheric warming.

- **Tropical Tropospheric Warming (TTW):** as in the TUW, but the equilibrium potential temperature change is independent of height within the tropopause,

$$F(\phi, p) = A$$

Here $A = -0.1438$ is the same as in the TUW. When the width of the heating $\phi_0$ is $10^\circ$, this value corresponds to 1.5K global mean potential temperature increase. As
shown later in this paper, the zonal mean midlatitude wind response to the narrow
warming in this configuration is similar to that of TTG, while the response to the
broad warming resembles that of TUW.

b. Zonally symmetric model

The response to the prescribed thermal forcing without eddy feedbacks is obtained by
making use of the zonally symmetric version of the model. In the zonally symmetric con-
figuration, only the zonal mean part (spectral zonal wave number 0) is integrated forward
in time. The contributions from eddies in the full model are added as external forcing to
the primitive equations, so that the zonally symmetric model yields the same time mean
circulation as does the full model.

The eddy forcing in the zonally symmetric model is derived from the full model and it
can be illustrated by expressing the primitive equations in a matrix form

\[
\frac{\partial \mathbf{x}}{\partial t} + \mathcal{L}(\mathbf{x}) + \mathcal{N}(\mathbf{x}) = \mathcal{D}(\mathbf{x})
\]  

(6)

where \(\mathbf{x}\) is a vector representing the prognostic variables in the model, including zonal wind,
meridional wind, temperature and surface pressure, \(\mathcal{L}()\) and \(\mathcal{N}()\) denote the linear and
nonlinear (e.g., advection term) operators. \(\mathcal{D}()\) represents the physical processes such as
friction and diabatic processes. In the Held-Suarez forcing, they are parameterized as linear
functions. Using the notation \(\mathbf{x} = \bar{\mathbf{x}} + \mathbf{x}'\), where the bar denotes the zonal mean and the
prime denotes the deviation from the zonal mean, taking the zonal mean and time mean of
(6), we can obtain

\[
\mathcal{L}(\bar{\mathbf{x}}) + \mathcal{N}(\bar{\mathbf{x}}) - \mathcal{D}(\bar{\mathbf{x}}) = -\mathcal{N}(\mathbf{x}') = F_{\text{eddy}}
\]  

(7)
where $F_{\text{eddy}}$ is the time and zonally averaged eddy forcing corresponding to the time and zonal mean fields $\bar{x}$. Furthermore, if we integrate the zonally symmetric model forward by one time step with the mean field, $\bar{x}$, we can obtain the tendency as

$$\frac{\partial \bar{x}}{\partial t} = -(\mathcal{L}(\bar{x}) + \mathcal{N}(\bar{x}) - \mathcal{D}(\bar{x})) \quad (8)$$

In comparison with equation (7), the negative tendency, $-\frac{\partial \bar{x}}{\partial t}$, is equal to the eddy forcing that maintain the zonal mean state, $\bar{x}$ in the original full model simulation. The details for this type of zonally symmetric model are also discussed in the appendix of Kushner and Polvani (2004).

c. Equilibrium and transient runs

The atmospheric responses to tropical warming are investigated using both equilibrium and transient simulations. In the equilibrated experiments, the perturbed experiments are run for 4000 days with each of thermal perturbations. For the selected thermal forcing of interest, we also run 100-member ensembles of transient experiments with a sudden switch-on of forcing. These transient experiments can help to understand how the circulation evolves gradually from the control climate to the perturbed climate. To this end, we first run the control experiment for 7000 days, of which from every 50 days of the last 5000 days, 100 realizations branch out to generate an ensemble. For each of the 100 realizations, the thermal perturbation is switched on instantaneously, and the model integrates for 200 days towards a new equilibrium. Additionally, the equilibrium and transient experiments are performed in the zonally symmetric model for the same sets of thermal perturbations. As the zonally symmetric model consists of no eddies, 1000-day integration is long enough
to reach equilibrium. Only one realization of transient experiment is needed for the zonally symmetric model to capture the temporal response to external thermal forcings.

3. Equilibrium responses

In this section, we show that the seemingly paradoxical results as reviewed in Introduction can be reconciled in a single dynamical model framework. We first show a TTG ($\phi_0 = 20^\circ$) experiment similar to Chang (1995), that is, a heating within 10 deg lat around the equator and cooling off equator between $10^\circ$ and $20^\circ$ latitudes. The left panels of Fig. 1 shows the changes in equilibrium temperature (1st row) and the circulation responses in overturning mass streamfunction (2nd row), temperature (3rd row), and zonal mean zonal wind (4th row). The enhanced diabatic heating gradient results in a strengthening and equatorward contraction of the Hadley cell. The temperature response is characterized by a weak uniform warming in the tropics, a cooling in the mid-latitudes between 25-45$^\circ$, and a warming further poleward. The zonal wind response consists of a barotropic dipole straddling the core of the climatological jet so that the jet stream shifts equatorward as a result. All the features described above are typical of the tropospheric response to El Niño (Seager et al. 2003; Lu et al. 2008) and in concert with the result of Chang (1995).

The TTG experiment is compared with a TUW ($\phi_0 = 30^\circ$) experiment, similar to the configuration used in Butler et al. (2010), on the right panels of Fig. 1. In contrast to the TTG, the circulation response to TUW leads to a somewhat opposite picture in the mid-latitudes. The Hadley cell weakens and expands poleward. The zonal wind also shifts poleward, in accordance with the poleward shift of the Ferrell cell. The temperature in this
TUW experiment exhibits an opposite pattern to that in the TTG case (cf. Figs. 1g and 1c) except over the regions of the direct impact of the tropical forcing. These results again, agree well with the similar case of Butler et al. (2010) (see their Fig. 2).

To verify the robustness of the circulation shift in the TTG and TUW experiments, we further perform sensitivity experiments in which the edge of the tropical heating is systematically broadened from 10°N/S to 30°N/S with an increment of 5° (Fig. 3). As the boundary of the subtropical cooling in the TTG gradually extends poleward, the TTG forcing tends to produce a greater shift in the jet. In the TTG group, although the westerly jet tends to shift equatorward with the intensity of the simulated Hadley cell, there is no simple linear proportionality between the two (Fig. 3(b)). Next, consistent with Butler et al. (2010), even the narrowest TUW forcing can produce a weak poleward shift of the eddy-driven jet (see Fig. 3(a), triangles), while the shifts forced by the broader TUW forcings (≥ 20°) are much more pronounced. Also, the TUW always weakens the Hadley cell, with a weaker intensity accompanying a more poleward jet shift (Fig. 3(b), triangles). In both sets of experiments examined, there is no change in the sign of the circulation shift. In summary, our modeling results agree with Chang (1995) and Butler et al. (2010): stronger tropical warming gradient generates a narrowed and strengthened Hadley cell and tropical upper tropospheric warming results in a broadened and weakened Hadley cell.

Furthermore, when the tropical heating is prescribed throughout the troposphere with the TTW forcings, we see an equatorward-to-poleward transition as the breadth of the tropical heating is systematically expanded from 10°N/S to 30°N/S, with the transition point occurring around φ₀ = 17.5°N/S (Fig. 3(a)). Interestingly, the circulation change with TTW exhibits a monotonic sensitivity to the meridional extent of the heating, with a broader
heating driving a broader Hadley cell extent and a more poleward eddy-driven jet and a lesser
intensification of the Hadley cell (circles in Fig. 3). In contrast to the axis-symmetric theory
(Held and Hou 1980), the intensity of the Hadley cell is more sensitive to the heating gradient
within the tropics than the overall gradient; the extent of the Hadley cell does not have a
simple proportional relationship with the equator-to-pole temperature gradient, either.

Fig. 2 illustrates the responses in circulation for narrow \((\phi_0 = 10^\circ)\), left panels) and broad
\((\phi_0 = 30^\circ)\), right panels) TTW forcings. Both cases are characterized by a strengthening of
the Hadley cell and a warming in the tropics. The extratropical responses, however, are
distinct in both the latitudinal shifts of the westerly jet and the overturning circulation. The extratropical differences between the two cases resemble qualitatively those between the
TTG and the TUW experiments in Fig. 1 or those between El Niño and global warming
(Lu et al. 2008). Thus these two TTW experiments with differing tropical warming widths
provide a pair of analogues in a dry dynamical framework corresponding to El Niño and global
warming, respectively. In the following sections, the narrow versus broad TTW experiments
will be analyzed in details to probe into their underlying dynamics.

Can the distinct extratropical circulation changes between narrow and broad warmings
be obtained merely through the zonally symmetric dynamics? We apply the same narrow
versus broad TTW forcings as illustrated in the top panels in Fig. 2 to the zonally symmetric
model, wherein the eddy forcing is fixed to be the same as in the control run and thus not
allowed to respond to the thermal perturbation. Interestingly, both the narrow and broad
TTW forcings drive characteristically similar patterns of tropical warming and produce an
equatorward enhancement of the westerly jet (Fig. 4). This result pinpoints unequivocally
the indispensable roles of the eddy feedbacks in producing the opposite equilibrated shifts of
the westerly jet between the two cases. As both the zonal wind and PV gradient are altered on the climatological jet’s equatorward flank, the conventional linear wave theory would predict an equatorward shift of the zonal jet for both cases through either wave refraction (Seager et al. 2003) or shift of the critical latitudes (Lu et al. 2008). Therefore, the small-amplitude linear theories are inadequate to explain the distinct full model responses between the two cases, posing a challenge for us to unravel the contrasting mechanisms between the El Niño-like and global warming-like circulation changes. This motivates us to perform large size ensemble experiments with an abrupt switch-on of the thermal forcing for both cases and diagnose the day-to-day adjustment processes using a novel, finite-amplitude wave activity budget.

4. Transient responses to narrow versus broad tropical warming

To distinguish the mechanisms responsible for the opposite shift of the eddy-driven circulation between the narrow and the broad TTW cases, we resort to transient ensemble experiments, in which the thermal forcing is abruptly switched on and kept constant. The ensemble size is 100, but by averaging both hemispheres the actual sample size for the ensemble mean is 200. Note that even with the sample size of 200, the internal noise remains sizable in the ensemble mean, due to the vibrant internal variability of the midlatitude winds. The following investigation will be focused only on the forced ensemble mean features.

The top panels of Fig. 5 show the evolutions of the anomalous zonal wind at 875-hPa in
the transient experiments for the narrow (left) and broad (right) warmings corresponding to
the equilibrium solutions in Fig. 2. Initially for both cases, positive zonal wind anomalies
appear between 20° - 40° latitude, indicating an equatorward enhancement or shift of the
eddy-driven westerly jet. While the anomalies in the narrow warming case grow in intensity
with little structural change and quickly reach equilibrium at around day 40, the positive
anomalies in the broad warming case gradually shift poleward, undergoing a transition from
an equatorward enhancement to a poleward shift. It takes more than 100 days for the
ensemble mean response to reach equilibrium. The upper tropospheric winds show similar
evolutions but with larger magnitudes (not shown). We notice that the temporal evolution in
the broad warming case is somewhat different from the similar ensemble transient simulation
of Butler et al. (2011), wherein the zonal wind anomalies show little shift during the transient
adjustment. This may be attributed to their small ensemble size (24 realizations), rendering
the small initial changes hard to detect. The initial equatorward shifts we see in our model
are also consistent with the thermally-driven changes simulated in the zonally symmetric
model.

Similar transient experiments are also performed using the zonally symmetric version of
the model. Consistent with the equilibrated results in Fig. 4, the evolutions of the zonal
winds for the narrow and broad warmings are similar, both characterized by a monotonic
increase of the zonal wind in the subtropical upper troposphere. Thus, in the absence of
eddy feedback there is little change in the extratropics.

From the perspective of angular momentum balance, the midlatitude barotropic wind
(with the time tendency term neglected) can be roughly thought of as the result of the
balance between the eddy momentum convergence and the surface friction (e.g., Chen et al.
2007), which may be parameterized as a linear function of the near-surface zonal wind. Indeed, as shown in the middle panels of Fig. 5 the evolution of the vertically integrated eddy momentum convergence displays rather similar temporal structure to the near-surface wind. The dominance of the eddy momentum forcing is further confirmed by a direct calculation of the momentum advection by the zonal mean wind components: the latter in the extratropics is an order of magnitude smaller than the former, as expected from the quasi-geostrophic scaling.

The lead-lag relationship between the phases of the eddy momentum forcing and the near-surface zonal wind is clearer when both are projected onto the pattern of the leading mode (i.e., the annular mode or zonal index) of the 875 hPa zonal wind (bottom panels of Fig. 5). On the one hand, for the narrow warming case, both projections show a monotonic acceleration with time towards the equilibrium state during the adjustment phase. No distinct phase lag or lead can be discerned between the eddy forcing and the 875 hPa zonal wind. On the other hand, for the broad warming case, consistent with the upper right panel of Fig. 5, the time series of the zonal index descends first (from day 1 to day 13) and then ascends afterwards. As a result, the zonal index undergoes a negative-to-positive transition at approximately day 38. It is important to note that the transition of the eddy momentum forcing takes place much earlier and it is the positive projection of the eddy momentum forcing that is responsible for the upward swing of the zonal index between days 13-38.

In observing the similar tendencies of the evolutions between the narrow and broad warming cases during the first 13 days and the intriguing transition in the latter case, we divide the transient adjustments of the two ensemble experiments into 3 different stages: stage-I (days 1-13) wherein the eddy-driven jet shifts equatorward in both warming cases; stage-
II (days 13-38) wherein the zonal index starts to turn around toward the transition point under broad warming, while it continues to grow in the same direction set by the initial stage under narrow warming; stage-III (day 38 onward): the circulation is nearly in equilibrium for the narrow warming case, while the zonal index just crosses the transition point and starts ramping up towards a positive equilibrium. With the aid of this categorization, we may begin to address the issue with respect to the cause for the poleward shift of the westerly jet in the broad warming case.

5. Mechanisms of the tropospheric jet shift

The experiments and analyses above point unequivocally to the importance of the eddy adjustment in the poleward shift of the eddy-driven jet under broad tropical heating. Here, we strive to elucidate the mechanisms of eddy adjustment by the Eliassen-Palm (E-P) flux diagnostics, maximum Eady growth rate analysis, and a finite-amplitude wave activity budget.

a. E-P flux

Starting with the conventional Transformed Eulerian Mean (TEM) momentum equation (Edmon et al. 1980; Andrews et al. 1987), we have

\[ \frac{\partial \tilde{u}}{\partial t} = f \tilde{v}^* + \frac{1}{a \cos \phi} \nabla \cdot \vec{F} + \bar{X} \]  (9)
where $f$ is the Coriolis parameter, $v^*$ is the residual meridional velocity, $X$ is the friction near the surface, $\nabla \cdot \vec{F}$ is the E-P divergence, which has the form of

$$
\frac{1}{a \cos \phi} \nabla \cdot \vec{F} = -\frac{1}{a \cos^2 \phi} \frac{\partial (v' u' \cos^2 \phi)}{\partial \phi} + f \left( \frac{\nu' \theta'}{\partial p} \right)_p
$$

where $\Theta$ is the hemispheric mean of potential temperature $\theta$. Assuming that eddies are geostrophic and ignoring the divergence associated with the gradient in $f$, it is readily shown that

$$
\overline{v'q'} = \frac{1}{a \cos \phi} \nabla \cdot \vec{F}
$$

where the left hand side is the zonal mean eddy PV flux, and $q$ is the quasi-geostrophic (QG) potential vorticity,

$$
q = f + \zeta + f \frac{\partial}{\partial p} \left( \frac{\theta - \Theta}{\partial \phi} \right)
$$

Fig. 6(a) shows the climatological E-P flux (vectors) and divergence (shading), together with the momentum convergence (contours). The E-P vectors represent the propagation of the QG wave activity and are typically upwards in midlatitude, veering equatorward aloft. The vertical component of the E-P flux arises from the meridional heat flux and acts to reduce the intensity of the mid-latitude westerlies aloft, transferring angular momentum from the upper troposphere to the surface to be finally dissipated by the surface friction. The horizontal component of the E-P flux acts to extract angular momentum from the subtropics and deposit in the mid-latitudes, so as to produce a westerly jet.

Under a sudden switch-on of the narrow tropical heating, the upward E-P flux is enhanced during stage-I (from day 1 to day 13), giving rise to a convergence in the upper troposphere and a divergence in the lower troposphere (Fig. 6(b)). The anomalous upward E-P flux has a maximum at the equatorial flank of the climatological E-P flux maximum (and so the
climatological jet), thus shifting equatorward the total distribution of the E-P flux and the
momentum flux convergence/divergence. As such, the eddy-induced equatorward shift takes
place immediately during the first phase of the transient adjustment. The adjustment of
stage-II (day 13 to 38) is characterized by a continuation of the enhancement of the upward
wave activity propagation in the subtropics and a reduction poleward of it (Fig. 6(c)).
The resultant momentum convergence/divergence reinforces the original structure resulted
from stage-I and drives the equatorward shift of the eddy-driven jet towards its equilibrium
state. The tripolar structure of the eddy-induced momentum convergence/divergence is
also consistent with the change diagnosed with the space-time co-spectrum discussed in
Chen et al. (2008), implicative of the involvement of the interaction between the enhanced
subtropical wind and the equatorward propagation of the wave activity. Note, however,
that a very similar interaction also takes place during the first stage of the adjustment to
the broad warming as well. We argue that it is the sustaining baroclinicity at the latitudes
of 25°-30° imposed by the narrow tropical warming that provides the source of the wave
activity and underpins the upper level horizontal divergence of the wave activity flux. It is
thus remarkable to see next for the broad warming case that the eddy momentum forcing
begins to shift the jet poleward despite the lower tropospheric baroclinicity remains at the
equatorward side of the jet.

Fig. 6(d) shows the changes of the E-P flux and momentum convergence during the
first 13 days for the broad warming case. The adjustment during stage-I bears a consider-
able resemblance to its counterpart of the narrow warming case: an overall enhancement of
the upward wave activity flux, the upper tropospheric convergence and lower tropospheric
divergence of the E-P flux. The magnitude for the broad warming case is larger and the
impacts are more widely spread, thus projecting more strongly on the climatological flux and
cconvergence. Interestingly, the corresponding anomalous momentum convergence is similar
to the narrow warming case in both structure and magnitude and consequently exerts a
similar negative projection onto the annular mode (cf. Fig. 5 bottom). From day 13 to
38 (stage-II), the enhancement of the vertical E-P flux starts to attenuate, but unevenly in
space with the least reduction near 50° lat. Thus, relative to the control state, the maximum
of the anomalous upward E-P flux still takes place under the center of the eddy momentum
acceleration (Fig. 6(e)). One may be attempted to conjecture that it is the change of the
lower tropospheric baroclinicity that maintains the larger reduction of the wave production
at the equatorward side of the jet, and that the upper level momentum convergence change
should be attributed to the change of the lower level baroclinicity as in the narrow warm-
ing case. However, a closer inspection on the lower tropospheric baroclinicity suggests the
opposite, as to be elaborated in the next section.

b. Lower tropospheric baroclinicity analysis

The lower level baroclinicity can be measured by the maximum Eady grow rate (MEGR)
(see also Vallis 2006; Yin 2005; Brayshaw et al. 2008):

$$\sigma = -\frac{0.31 g}{N \Theta_0} \frac{1}{a} \frac{\partial \tilde{\theta}}{\partial \phi}$$

where $g$ is the gravitational acceleration rate, $N$ is Brunt-Väisälä frequency, $\Theta_0$ is the global
mean potential temperature, and $\tilde{\theta}$ is the zonal-mean potential temperature. The climato-
logical MEGR in the troposphere peaks at 42°N/S (not shown), consistent with the E-P
diagnosis (Fig. 6(a)). The left panels of Fig. 7 show the evolution of the anomalous MEGR
of the full-model and zonally-symmetric-model adjustment respectively in the narrow warming case. In the former, the positive anomaly first appears near 20° in the subtropics, but quickly weakens. In comparison with its zonally-symmetric-model counterpart (Fig. 7(b)), one can see that this anomaly is associated with the tropical adjustment to the imposition of the narrow heating in the absence of eddy feedbacks and it is quickly wiped out as the eddy adjustment begins to dominate. In comparison with the time series of the annular mode projections (Fig. 5 bottom), the development of a positive (negative) MEGR anomaly at the equatorward (poleward) flank of the jet coincides with the growth of the negative projection of the eddy momentum forcing and the negative zonal index during the transient adjustment. The synchronous evolution of all the three renders a simple interpretation of the response in the narrow warming case as discussed above.

For the broad warming case, during the first stage (≤ day 13) of the adjustment an anomalous baroclinicity first occurs at the equatorward flank of the jet, in concert with the eddy momentum acceleration at the similar latitudes associated with the equatorward enhancement of the E-P flux. The poleward shift of the momentum forcing during the second stage between day 13 and day 38, however, cannot be explained by the lower-level baroclinicity change, which remains at the equatorward side of the jet throughout this period. Further, the projection of the eddy momentum forcing onto the annular mode tends to lead the projection of the baroclinicity (by comparing the right panels of Fig. 5 with Fig. 7(c)). Thus, one cannot use the baroclinicity argument to explain the rise of the momentum forcing for the jet shift in the broad warming case. Instead, as will be shown in the next section, it is the irreversible wave breaking in the upper troposphere that holds the key to understanding the origin of the poleward shift in momentum flux during stage II. During stage III, a
poleward shift in the lower level baroclinicity emerges and grows in intensity. Note that
stage III is defined by the period when the eddy-driven jet is more poleward relative to the
climatological jet. This suggests as the jet starts to move poleward in stage III, this poleward
shift is further reinforced by the changes in the lower level baroclinicity, which provides a
positive baroclinic feedback (e.g., Robinson 2000; Chen and Plumb 2009). A similar MEGR
analysis is also performed for the corresponding adjustment in the zonally symmetric model.
In the absence of eddy feedbacks, the zonally symmetric Hadley cell adjustment fixates the
anomalous MEGR at 30° in the subtropics with little displacement throughout the evolution.
The resultant MEGR anomalies are located always more equatorward relative to the full
adjustment, highlighting the consequential role of eddies in shaping the thermal structure of
the midlatitude atmosphere.

c. Finite-amplitude wave activity diagnostics

We adopt a diagnostic framework developed by Nakamura and collaborators (Nakamura
and Zhu 2010; Nakamura and Solomon 2010) by combining the conventional Transformed
Eulerian Mean (TEM) momentum equation with the finite-amplitude wave activity budget.
Through relationship (11) the TEM momentum equation (9) is coupled with the pseudo-
momentum equation (e.g., Vallis 2006).

\[
\frac{\partial A}{\partial t} + \tilde{v}'q' = D
\]  

(14)

where, \( A = \frac{1}{2} (1_{\text{a}} \frac{\partial q}{\partial \phi})^{-1} q'^2 \) is wave activity, which is negative of pseudo-momentum, the second
term is the Eulerian mean eddy PV flux, and \( D \) represents the irreversible eddy mixing
processes. In eqn. (14), the sinks and sources of wave activity due to diabatic processes are
ignored, for they contribute little to the budget in the upper troposphere during the transition phase of the adjustment. Note that in the Eulerian formalism of the pseudo-momentum equation, the cubic and higher order terms in eddy amplitude have been neglected. Moreover, since the zonal mean PV gradient can change sign in the interior of the atmosphere and $A$ becomes unrealistically large as the PV gradient approaches zero, the small-amplitude formulation has a rather limited applications for studying midlatitude eddy mean-flow interaction.

Recently, by taking a Lagrangian approach in estimating the wave activity and introducing equivalent latitude $\phi_e$, Nakamura and collaborators (Nakamura and Zhu 2010; Nakamura and Solomon 2010) extended the concept of the wave activity to finite-amplitude waves and define finite-amplitude wave activity as

$$A(p, \phi_e) = \frac{1}{2\pi a \cos \phi_e} \left[ \int \int_{q > Q; \phi < \pi/2} q_{yy} dS - \int \int_{\phi_e < \phi < \pi/2} q_{yy} dS \right]$$

(15)

which measures the net displacement the PV contours from zonal symmetry. The equivalent latitude for the PV value $Q$ is determined by the requirement that the area enclosed by the PV contour towards the polar cap equals the area poleward of $\phi_e$

$$\int \int_{q > Q; \phi < \pi/2} dS = \int \int_{\phi_e < \phi < \pi/2} dS = 2\pi a^2 (1 - \sin \phi_e)$$

(16)

As such, there is a monotonic relationship between $\phi_e$ and $Q$ for both hemispheres. More importantly, the gradient of $Q$ with respect to $\phi_e$ is always positive. $A$ is positive definite and it assumes the familiar form of wave activity ($A = \frac{1}{2} (\frac{1}{a} \frac{\partial q}{\partial \phi})^{-1} q^2$) for the small-amplitude limit.

It is shown in Nakamura and Zhu (2010) that with the wave activity defined as above equation (15), the non-acceleration theorem holds accurately for both small-amplitude and
finite-amplitude eddies for inviscid, adiabatic fluid. Furthermore, within this hybrid Eulerian-
Lagrangian framework, the eddy dissipation term can be further formulated as to be propor-
tional to the Lagrangian PV gradient. Thus, equation (14) can be written as

$$\frac{\partial A}{\partial t} + v'q' \approx -\frac{K_{eff}}{a} \frac{\partial Q}{\partial \phi_e}$$

(17)

where $K_{eff}$ is the effective diffusivity of the irreversible mixing of PV across material sur-
faces (e.g., Nakamura 1996). In this paper, $K_{eff}$ can be either estimated indirectly as a
residual term ($D$) in equation (17) or calculated directly using the modified formalism for
the hyperdiffusion used in our spectral dynamical core (see Appendix A).

Making use of the relationship (11) under QG dynamics, we yield a budget equation for
the EP flux divergence:

$$v'q' = \frac{1}{a \cos \phi} \nabla \cdot \vec{F} \approx -\frac{\partial A}{\partial t} - \frac{K_{eff}}{a} \frac{\partial Q}{\partial \phi_e}$$

(18)

Equation (18) states that the E-P flux divergence/convergence, which serves as a momentum
source/sink for the zonal momentum equation (9), can be accounted for by the wave activity
tendency and by the irreversible wave dissipation. It is important to note that this is a hybrid
framework, in which the E-P flux divergence is local in latitude, but the wave activity and
PV gradient are evaluated following the material surface of the PV contours. The Eulerian
and Lagrangian components of the diagnosis are aligned together by matching the equivalent
latitudes of $A$ and $Q$ and the latitude grids for $v'q'$. Caution should be used in interpreting
the budget within the deep tropics, where the QG approximations are no longer valid.

In view of the fact that the poleward transition of the eddy momentum forcing during
Stage II in the broad warming case cannot be accounted for the lower tropospheric baroclin-
icity, we now turn our attention to the upper tropospheric wave dynamics, focusing on the
effects of the irreversible wave dissipation. According to eqn. (18), the change of the E-P flux divergence or eddy PV flux during the transition (Fig. 8(a)) can be attributed to the change of the negative tendency of the finite-amplitude wave activity (-dA/dt, Fig. 8(b)) and the change of the eddy dissipation (Fig. 8(c)). One can see that most of the PV flux change is explained by the change in dissipation. We further decompose the total dissipation into the component due to PV gradient change and that due to effective diffusivity change. We find that the enhanced dissipation (Fig. 8(c)) and hence the eddy PV flux convergence between 35 – 40° at 300 hPa level (Fig. 8(a)) result mostly from the increase of the effective diffusivity (Fig. 8(g)), while the reduction of the dissipation poleward of 40° and the associated eddy PV flux divergence are mostly attributable to the decrease of PV gradient (Fig. 8(d)(f)). Fig. 8(g) shows the change of the direct estimate of the effective diffusivity following the formalism of Nakamura and Zhu (2010) for a tracer subject to hyperdiffusion (see Appendix): the effective diffusivity increases at the poleward and upward portion of the mean subtropical maximum as well as over the extratropical surf zone. This former increase may have augmented the mixing at the equatorward side of the climatological mixing barrier and pushed the upper level PV gradient maximum poleward (Fig. 8(f)), while the latter increase aids in mixing the extratropical PV so as to reduce its gradient there.

In summary, the transient adjustments to narrow versus broad tropical warming can be understood as follows. Under the narrow warming, we see an increased baroclinicity in the subtropics initially. This low level baroclinicity is sustained throughout the transient adjustment, which initializes and maintains an equatorward jet shift. Under the broad warming, the circulation change may be interpreted as the following chain of reasoning in three stages. In Stage I, the zonally symmetric Hadley circulation adjustment to the
broad tropical warming shifts the subtropical jet equatorward and the baroclinic eddies become more vigorous as a result of increased subtropical baroclinicity. In stage II, stronger baroclinic eddies enhance the irreversible mixing and hence the dissipation of wave activity in the subtropical upper troposphere, while the enhanced diffusivity at high latitudes reduces the PV gradient, resulting in a weaker wave activity dissipation poleward of the climatological jet. These changes in wave activity dissipation are accompanied by increased equatorward wave propagation and poleward eddy momentum flux. In Stage III, a poleward shift in circulation is associated with a shift in the lower level baroclinicity, the latter providing a positive feedback to the initial poleward drift of eddy-driven winds.

One may notice some resemblance of our transient response to the broad heating case to the upper tropical tropospheric warming case studied in Butler et al. (2011). But our interpretation differs from Butler et al. (2011) in a fundamental way. In Butler et al. (2011), the evolution of the eddy-driven jet is characterized by a monotonic, down-gradient eddy PV flux adjustment to the change of the isentropic slope and the associated PV gradient. To the extent that the change of the effective diffusivity can be ignored, the eddy PV flux (or E-P flux convergence) response is largely down the gradient of the PV as in Butler et al. (2011). However, in our case, the effect of the changing $K_{eff}$ is the key to the reversal of the eddy PV flux divergence in the subtropical upper troposphere. Nakamura and Zhu (2010) showed that ignoring the slow thermal forcing, the daily variability the PV gradient can be written as

$$\frac{\partial}{\partial t} \left( \frac{1}{a} \frac{\partial Q}{\partial \phi_e} \right) \approx \frac{1}{a^2 \cos \phi_e} \frac{\partial^2}{\partial \phi_e^2} \left( K_{eff} \cos \phi_e \frac{1}{a} \frac{\partial Q}{\partial \phi_e} \right)$$

(19)

If we identify the jet by the maximum upper level PV gradient, the change of the effective
eddy diffusivity may be the single most important factor in shaping the profile of the upper level PV: a poleward advance of the subtropical surf zone should push the maximum PV gradient and hence the jet poleward.

6. Discussion and conclusion

In an attempt to address the apparent paradox regarding the different responses in the zonal mean circulation to the narrow El Niño-like forcing versus the broad global warming-like forcing, a sensitivity study is carried out with regards to the width, the vertical level, and the meridional gradient of the tropical heating using a dry atmospheric dynamical model. The results suggest that all the previous studies can be reconcilable within a single modeling system. With a tropical heating comprising an equatorial heating and a subtropical cooling so as to increase tropical temperature gradient (TTG) (resembling the heating profile under El Niño condition), this dry model produces a narrowed and strengthened overturning circulation and an equatorward shift of the eddy-driven jet, corroborating the pioneering work of Chang (1995). With the tropical upper-tropospheric warming (TUW), the model also replicates the robust poleward shift of the eddy-driven jet regardless the width of the heating, as found in Butler et al. (2010). Considerable linearity is found between the width of the tropical heating and shift of the jet and between the intensity and the poleward extent of the Hadley cell, whereas an intensification of the Hadley cell does not necessarily lead to an expansion.

Additionally, in one set of experiments with the tropical warming imposed throughout the troposphere, the Hadley cell intensity increases for all widths of the heating, but the
shift of the jet undergoes an equatorward-to-poleward transition as the meridional breadth
of the heating gradually widens. The opposite shift in circulation for narrow versus broad
warming in this set of experiments is analyzed in details. In keeping with previous studies
(e.g., Brayshaw et al. 2008; Chen et al. 2010), we also find in the equilibrium response of
our sensitivity experiments that the locations of the eddy-driven jet coincide with those of
lower level baroclinicity. However, this does not necessarily imply causality between the
baroclinicity and the jet shift. The transition towards a poleward jet shift in the broad
tropical tropospheric warming case provides a counter-argument for the causality between
the baroclinicity and eddy-driven wind. In the time sequences of the transient adjustment
to the broad tropical heating, the lower level baroclinicity actually increases first at the
equatorward side of the jet and the eddy momentum acceleration at the poleward flank
of the jet leads the poleward shift of the baroclinicity. Since the cause must occur before
its effect in time, processes other than the change of baroclinicity must be responsible for
initiating the jet shift.

By looking into the wave activity budget during the transition in the broad warming
case, we find that the dissipation of the wave activity is enhanced at the equatorward side
of the mean jet due to an increase of effective diffusivity. On the other hand, the dissipation
is reduced at middle latitudes due largely to the decrease of the PV gradient. The increased
dissipation at the equatorward side of the jet and the decreased dissipation at higher lati-
tudes serve as a pair of anomalous wave sink and source respectively, maintaining anomalous
equatorward wave activity propagation during the transition phase of the adjustment. We
thus argue that it is the irreversible wave mixing processes that instigates the eddy momen-
tum forcing for the poleward jet transition, in accord with the perspective of wave breaking
proposed by Rivi`ere (2011). The eventual poleward movement of the baroclinicity may be understood in terms of baroclinic feedback mechanism whereby the upper level wave drag induces positive baroclinicity below in the direction of the jet shift (e.g., Robinson 2000).

Generally speaking, in response to tropical diabatic heating, both the upper tropospheric wave breaking and the lower level baroclinicity are important in controlling the position of the jet stream and the terminus of the Hadley cell. The final destination of the eddy-driven jet can be thought of as the result of the tug of war between the wave dissipation in the upper troposphere and the change of baroclinicity in the lower troposphere. For the broad TTW experiments, in spite of the initial increase of the subtropical baroclinicity, it is dominated by the irreversible upper tropospheric wave breaking during the poleward transition. But for the narrow TTW and all the TTG cases, the anomalous baroclinicity gains a foothold at the equatorward side of the jet and dominates the final outcome of the competition.
The direct estimate of effective diffusivity

From the appendix of Nakamura and Zhu (2010), for the m-th order hyperdiffusion, the effective diffusivity can be directly diagnosed as

\[
K_{\text{eff}} = \langle k_2^m (-1)^{m-1} \nabla [\nabla^2_q (m-1) q] \cdot \nabla q \rangle_Q
\]

where \( k_2^m = 4.2 \times 10^{38} \) m^8 s^-1 is the diffusion coefficient for the 8-order hyperdiffusion (m=4) used in our spectral dynamical core. This diffusion coefficient damps the highest-order harmonic with an e-folding time of 0.1 day. \( q \) is the QG-PV. \( \langle \cdot \rangle_Q \) denotes the area-weighted average around the \( Q \) contour. \( \phi_e \) is the equivalent latitude. For the regular diffusion m=1, we obtain the conventional formulation of the effective diffusivity as in Nakamura (1996).
REFERENCES


The descriptions and parameters for the three sets of thermal perturbations.

\[ T_{\text{eq}}^p(\phi, p) = \max\{200K, [315K - \delta_y(\sin^2 \phi + \mathcal{F}(\phi, p)W(\phi, \phi_0))] - \delta_z \log(p/p_0) \cos^2 \phi\} \]

\((p/p_0)^{\kappa}\) is the equilibrium temperature, where \(\delta_y=60K\), \(\delta_z=10K\). \(\mathcal{F}(\phi, p)W(\phi, \phi_0)\) determines the thermal forcing. For the TTG case, \(A\) varies with different width so that the global mean temperature keeps unchanged. For the other two warming cases, \(A\) is constant independent of the width of the warmings.

\(W(\phi, \phi_0) = 0.5(1 - \tanh((|\phi| - \phi_0)/\delta\phi))\) and \(\exp((p/p_s - 0.3)^2/(2 \cdot 0.11^2))\) are the meridional and vertical constraints for the thermal forcings, respectively. \(\phi_0\) is the transition latitude, determining the width of the thermal perturbation.

See the text in section 2a for details.
Tropical Temperature Gradient (TTG): heating in the deep tropics, and cooling in the subtropics; global mean temperature keeps unchanged. 

\[ F(\phi, p) = A + \sin^{1.25}(|\phi|) - \sin^2(\phi) \]

Tropical Upper-tropospheric Warming (TUW): tropical heating distributed in the upper troposphere. 

\[ F(\phi, p) = A \cdot \exp\left[\left(\frac{p}{p_s} - 0.3\right)^2 / (2 \cdot 0.11^2)\right] \]

Tropical Tropospheric Warming (TTW): vertically uniform potential temperature perturbation in the troposphere. 

\[ F(\phi, p) = A \]

<table>
<thead>
<tr>
<th>Experiment and description</th>
<th>A</th>
<th>( \phi_0 )</th>
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<tbody>
<tr>
<td>Tropical Temperature Gradient (TTG):</td>
<td>-0.0471</td>
<td>10°</td>
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<tr>
<td>Heating in the deep tropics, and cooling in the subtropics;</td>
<td>-0.0637</td>
<td>15°</td>
</tr>
<tr>
<td>global mean temperature keeps unchanged.</td>
<td>-0.0793</td>
<td>20°</td>
</tr>
<tr>
<td>( F(\phi, p) = A + \sin^{1.25}(</td>
<td>\phi</td>
<td>) - \sin^2(\phi) )</td>
</tr>
<tr>
<td></td>
<td>-0.1037</td>
<td>30°</td>
</tr>
<tr>
<td>Tropical Upper-tropospheric Warming (TUW):</td>
<td>-0.1438</td>
<td>10°</td>
</tr>
<tr>
<td>Tropical heating distributed in the upper troposphere.</td>
<td>-0.1438</td>
<td>15°</td>
</tr>
<tr>
<td>( F(\phi, p) = A \cdot \exp\left[\left(\frac{p}{p_s} - 0.3\right)^2 / (2 \cdot 0.11^2)\right] )</td>
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<tr>
<td></td>
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<td>Vertically uniform potential temperature perturbation in</td>
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<td>the troposphere. ( F(\phi, p) = A )</td>
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<td>-0.1438</td>
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**Table 1.** The descriptions and parameters for the three sets of thermal perturbations. 

\[ T_{eq}^p(\phi, p) = \max\{200K, [315K - \delta_y(\sin^2 \phi + \mathcal{F}(\phi, p)\mathcal{W}(\phi, \phi_0)) - \delta_z \log(\frac{p}{p_0}) \cos^2 \phi] \cdot (\frac{p}{p_0})^\kappa\} \]

is the equilibrium temperature, where \( \delta_y = 60K, \delta_z = 10K \). \( F(\phi, p)\mathcal{W}(\phi, \phi_0) \) determines the thermal forcing. For the TTG case, \( A \) varies with different width so that the global mean temperature keeps unchanged. For the other two warming cases, \( A \) is constant independent of the width of the warmings. \( W(\phi, \phi_0) = 0.5(1 - \tanh((|\phi| - \phi_0)/\delta_\phi)) \) and \( \exp((p/p_s - 0.3)^2 / (2 \cdot 0.11^2) \) are the meridional and vertical constraints for the thermal forcings, respectively. \( \phi_0 \) is the transition latitude, determining the width of the thermal perturbation. See the text in section 2a for details.
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2 Zonally averaged (contours) climatologies and (shading) equilibrated responses to the (left) narrow ($\phi_0 = 10^\circ$) and (right) broad ($\phi_0 = 30^\circ$) tropical tropospheric warming. (a)(e): equilibrium temperature in unit of K, (b)(f): meridional streamfunction in unit of $10^9$ kg s$^{-1}$, (c)(g): temperature in unit of K, (d)(h): zonal wind in unit of m s$^{-1}$.

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4 (Contour) Climatologies and (shading) equilibrated responses of temperature and zonal-wind to (left) the narrow and (right) broad tropical tropospheric warming in the zonally symmetric models, in which the same eddy forcings are derived from the control runs. The thermal perturbations are the same as those in Figure 2.
Top: the evolutions of zonal-mean zonal wind anomaly at 875 hPa in the transient experiments of (left) narrow and (right) broad warmings. Middle: similar to the top, but for the vertically averaged eddy momentum convergence anomaly. The black dashed lines in the top and middle panels denote the latitude of the latitude of the climatological jet. Bottom: similar to the top, but for the projection of the zonal wind and eddy momentum convergence onto the leading EOF of 875-hPa zonal wind. In the bottom panel, the left black dash-dot line indicates day 13, in which the eddy momentum convergence projection minimizes in the broad warming; the right black dash-dot line indicates day 38, in which the zonal wind projection transits from negative to positive in the broad warming.

(a): climatological E-P vector, (shading) E-P flux divergence, and (contours) eddy momentum convergence in the control experiments. The red solid contours denote the positive eddy momentum convergence and the black dashed contours denote the negative convergence. (b), (c): similar to (a), but for the ensemble mean anomalies of day 13, and changes from day 13 to 38 in the transient experiments under narrow warming. (d), (e): similar to (a), but for the ensemble mean anomalies of day 13, and changes from day 13 to 38 in the transient experiments with broad warming. The contour interval is 1 m s\(^{-1}\) day\(^{-1}\) in (a) and 0.4 m s\(^{-1}\) day\(^{-1}\) in (b)-(e).
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Fig. 6. (a): climatological E-P vector, (shading) E-P flux divergence, and (contours) eddy momentum convergence in the control experiments. The red solid contours denote the positive eddy momentum convergence and the black dashed contours denote the negative convergence. (b), (c): similar to (a), but for the ensemble mean anomalies of day 13, and changes from day 13 to 38 in the transient experiments under narrow warming. (d), (e): similar to (a), but for the ensemble mean anomalies of day 13, and changes from day 13 to 38 in the transient experiments with broad warming. The contour interval is 1 m s\(^{-1}\) day\(^{-1}\) in (a) and 0.4 m s\(^{-1}\) day\(^{-1}\) in (b)-(e).
Fig. 7. The evolutions of low-level maximum eady growth rate (MEGR) anomalies in the transient experiments of (left panel: (a)(b)) the narrow and (right panel: (c)(d)) broad warmings. The top panels show the MEGR adjustment in the full model and the bottom panels indicate the MEGR adjustment in the zonally symmetric model. The MEGR anomalies are averaged vertically from 500 hPa to 900 hPa, and in unit of $10^{-7}$ s$^{-1}$. The black dashed line denotes the latitude of the climatological jet. The contour interval is $6.0 \times 10^{-7}$ s$^{-1}$ in the full model (a)(c) and $1.2 \times 10^{-6}$ s$^{-1}$ in the zonally symmetric model (b)(d). The two dash-dot lines in (a)(c) indicate day 13 and day 38, respectively.
Fig. 8. The zonal and ensemble mean change of the budget (equation (18)) from day 13 to day 38 in the broad warming. (a) eddy PV flux, (b) negative wave activity tendency, (c) total dissipation change, (d) dissipation change due to PV gradient change, (e) dissipation change due to effective diffusivity change, (f) PV gradient change, and (g) direct estimate of the effective diffusivity change. The contours in (a),(c),(d),(e) are the climatological mean pv flux (equal to dissipation) with contour interval of 2 m s$^{-1}$ day$^{-1}$. The contours in (f) and (g) are climatological mean $\frac{\partial Q}{\partial \phi}$ (interval of $4 \times 10^{-11}$ m$^{-1}$s$^{-1}$) and $K_{eff}$ (interval of 4.0 $\times$ $10^5$ m$^2$ s$^{-1}$). Note all the panels are shown above 500 hPa.