Thermodynamic and Dynamic Mechanisms for Hydrological Cycle

Intensification over the Full Probability Distribution of Precipitation Events

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Precipitation changes in a warming climate have been examined with a focus on either mean precipitation or precipitation extremes, but changes in the full probability distribution of precipitation have not been well studied. This paper develops a methodology for quantile-conditional column moisture budget of the atmosphere for the full probability distribution of precipitation. Analysis is performed on idealized aquaplanet model simulations under 3K uniform SST warming across different horizontal resolutions. It is found that specific humidity and horizontal mass convergence in a given precipitation percentile range are weakly correlated and thus their conditional averages yield a clear separation between the moisture (thermodynamic) and circulation (dynamic) effects of vertical moisture transport on precipitation. The thermodynamic response to idealized climate warming can be understood as a generalized ‘wet-get-wetter’ mechanism that the heaviest precipitation of the probability distribution is enhanced most from increased gross moisture stratification, at a rate controlled by the change in lower tropospheric moisture rather than column moisture. The dynamic effect, in contrast, can be interpreted by shifts in large-scale atmospheric circulations such as the Hadley cell circulation or midlatitude storm tracks. Furthermore, horizontal moisture advection, albeit of secondary role, is important for regional precipitation change. In particular, the change in horizontal advection and mass convergence in the subtropics under uniform SST warming can offset the thermodynamic contribution to more extreme precipitation. Although similar mechanisms are at play for changes in extreme precipitation, increases in high percentiles of precipitation tend to be more widespread than increases in the mean, especially in the subtropics.
1. Introduction

Despite much progress in modeling the global hydrological cycle, it is still challenging for state-of-the-art climate models to reliably simulate the frequency, intensity, and spatial pattern of precipitation at regional scales in a warming climate (e.g. Dai et al. 1999; Trenberth et al. 2003; Dai and Trenberth 2004; Sun et al. 2006). From an energy perspective, global energy balance places a strong constraint on the global-mean rainfall, but the spatiotemporal pattern of precipitation on the regional scale and its response to climate warming are less constrained. Regional precipitation changes can be largely influenced by issues in traditional convective parameterizations and their interactions with the large-scale dynamics in the models. For example, it has been shown that a key factor in modulating the tropical precipitation response to global warming is the tightening of the ascending branch of the Hadley Circulation coupled with a decrease in tropical high cloud fraction (e.g. Su et al. 2014; Lau and Kim 2015; Su et al. 2017). In the extratropics, precipitation extremes are expected to occur more frequently on the poleward flank of midlatitude storm tracks under climate warming due to the poleward shift of storm tracks (e.g. Lu et al. 2014; Pfahl et al. 2017). Regional projections of precipitation can also be affected by the intricate interplay among aerosols, cloud, and large-scale circulation (e.g. Ming and Ramaswamy 2009; Ming et al. 2011; Chen et al. 2011). As the resolution of climate models begins to resolve important cloud processes, it is important to develop robust understanding of the spatiotemporal variability of precipitation in climate models across model resolutions.

It has been well recognized that a decomposition of the global hydrological cycle into thermodynamic and dynamic mechanisms is valuable for our understanding of the uncertainties in climate projection, because the thermodynamic component of climate change signal is more robust than its dynamic counterpart (e.g. Xie et al. 2014). On the one hand, the thermodynamic mechanism
can be attributed to the Clausius-Clapeyron (CC) relation and surface warming, which predicts
about 7% increase in atmospheric moisture abundance per K warming. Thus, a warmer climate with
no change in atmospheric circulation can result in a 'wet-get-wetter' mechanism, with enhanced
moisture flux leading to subtropical dry regions getting drier and tropical and midlatitude wet re-
gions getting wetter (e.g. Chou and Neelin 2004; Held and Soden 2006; Chou et al. 2009). On the
other hand, changes in atmospheric circulation can alter the geographic distribution of subtropical
dry zones and midlatitude storm tracks. For example, the moisture advection in a warming climate
can lead to an 'upped-ante' mechanism, with an equatorward contraction of tropical convective
margin owing to increased equatorward transport of subtropical dry air (Chou and Neelin 2004;
Chou et al. 2009). The poleward edge of the subtropical dry zone can move polewards associated
with the Hadley cell expansion (e.g. Lu et al. 2007) or midlatitude jet shift (e.g. Chen et al. 2008).
Particularly over the US west coast, the predicted wetting trend under global warming has been at-
tributed to an eastward extension of the North Pacific jet stream (Neelin et al. 2013), North Pacific
storm tracks (Chang et al. 2015), or a change in local stationary wave pattern (Seager et al. 2014;
Simpson et al. 2016).

Furthermore, precipitation extremes are expected to increase at a faster rate than mean precipita-
tion (e.g. Hennessy et al. 1997; Kharin and Zwiers 2000; Wehner 2004; Pall et al. 2006). Assuming
no change in the lower-level mass convergence, the CC relation provides a thermodynamic con-
straint on the heaviest rainfall events when the column water vapor is likely precipitated out (e.g.
Pall et al. 2006; Chou et al. 2012). Statistical analysis based on an empirical separation between the
changes in vertical velocity and moisture content (Emori and Brown 2005; Chen et al. 2011) has
identified consistent thermodynamic changes in spite of spatial differences in greenhouse gas and
aerosol forcings. More physics-based scaling analysis has found robust thermodynamic changes
in precipitation extremes on the global scale but more uncertain dynamic changes on the regional scales (O’Gorman and Schneider 2009; Pfahl et al. 2017).

While the mechanisms for mean precipitation and precipitation extremes have been separately studied, a consistent thermodynamic and dynamic explanation of the hydrological cycle is still lacking for the full precipitation distribution. The goal of this paper is to develop a robust thermodynamic and dynamic decomposition for the full probability distribution of precipitation events. This decomposition will be examined in idealized aquaplanet simulations with varied horizontal resolution subject to 3K uniform Sea Surface Temperature (SST) warming in this paper and then applied to the simulations of Community Earth System Model (CESM) large ensemble (LENS) with realistic climate change projections in a companion paper (Norris et al. 2018).

The paper is organized as follows. Section 2 will briefly introduce the idealized aquaplanet models used in this study. The formulation of the conditional column water vapor budget, its thermodynamic and dynamic decompositions, and the definition of gross moisture stratification are presented in section 3. In section 4, the moisture budget for mean and extreme precipitation is analyzed for aquaplanet simulations. Section 5 gives the thermodynamic and dynamic mechanisms for the full distribution of precipitation. Conclusion is provided in section 6.

2. Idealized aquaplanet simulations

The simulations examined in this study are a set of aquaplanet experiments with idealized SST boundary conditions using the hydrostatic version of the Model for Prediction Across Scales (MPAS). The atmospheric dynamical core of the MPAS is based on centroidal Voronoi tesselations with an option to run at variable resolution meshes for regional refinement (Ringler et al. 2008). The experiments are performed with quasi-uniform horizontal resolutions coupled with the physics parameterizations of the Community Atmosphere Model version 4 (CAM4) (Neale
and Coauthors 2010). The aquaplanet experiments are forced by a prescribed zonally symmetric SST profile with no sea ice, as proposed by Neale and Hoskins (2000). The SST distribution in the control simulation is specified as $\text{SST} = 27(1 - \sin^2(3\phi/2))^\circ C$ for the latitude range of $-\pi/3 < \phi < \pi/3$ and is $0^\circ C$ elsewhere. Idealized climate warming simulations are performed with 3K uniform SST warming. Readers are referred to previous studies (e.g. Skamarock et al. 2012; Park et al. 2013; Rauscher et al. 2013; Sakaguchi et al. 2015) on the detailed documentation of the MPAS model and the statistics of its aquaplanet simulations with different SST configurations. In particular, Yang et al. (2014b) have analyzed the sensitivity of extreme precipitation to horizontal resolution in these simulations using a moisture budget.

As we are concerned about the robustness of precipitation extremes across horizontal model resolutions, experiments are conducted at three different resolutions with the mesh size approximately equal to 240, 120, and 60km, respectively, for both the control and 3K warming simulations. Each simulation has been run for 3 years, with the last 2 years used for our analysis. The data analyzed are based on 6 hourly output. As the horizontal resolution of the MPAS model is gradually increased, all the adjustable parameters in the CAM4 physics suite are fixed at the standard values, except that the $\nabla^4$ horizontal diffusion coefficients are reduced to minimize the impact of the numerical diffusion on the model atmosphere without violating the criterion of numerical instability. See Lu et al. (2015) for an example of the dynamical convergence of midlatitude jet streams.

Because SSTs and radiative forcing agents are zonally symmetric, the statistics of precipitation and its underlying dynamics should be zonally symmetric in the long-term average. Taking advantage of this zonal symmetry, the analysis is performed by aggregating all the data along the same latitude such that 2 years of data are sufficient for our analysis. Furthermore, while the horizontal model resolution is systematically increased, the data are aggregated over the grid resolution of
the 240km run for better comparison across resolutions. The results for the 60km run without
aggregation are quantitatively similar but noisier at the fine scales.

As the precipitation distribution is highly skewed, we choose to divide the precipitation distri-
bution unevenly into $M$ percentile bins.

$$e_i = 100\left[1 - 10^{-3\left(\frac{i-0.5}{M-0.5}\right)}\right], \quad i = 1, \cdots, M$$  \hspace{1cm} (1)

The largest percentile value considered here is $e_M = 99.9$. The conditional mean at the $e_i$th per-
centile is evaluated as the average over the percentile range of $[e_{i-0.5}, e_{i+0.5}]$ so as to increase the
sample size for precipitation extremes. For example, the conditional mean onto the 99.9th per-
centile of precipitation for $M = 11$ bins is averaged over all the events between the 99.86th and
99.93th percentiles of precipitation. The 11 bins are used to ensure a balance between the sample
size in each bin and the resolution in probability distribution. Quantitatively similar results are
obtained when the number of bins is doubled.

3. The moisture budget over the full probability distribution of precipitation

a. Conditional column water vapor budget

The vertically integrated moisture budget of the atmosphere can be written as (e.g., Seager and
Henderson 2013)

$$\frac{\partial}{\partial t} \int_0^{p_s} q \frac{dp}{g} = -\nabla \cdot \int_0^{p_s} \mathbf{v} q \frac{dp}{g} - (P - E)$$  \hspace{1cm} (2)

where $t$ is time, $p$ is pressure, $p_s$ is surface pressure, $\mathbf{v} = (u, v)$ is the horizontal velocity vector,
$g$ is gravitational acceleration, and $q$ denotes specific humidity. $P$ and $E$ are precipitation and
evaporation rate at the surface in units of kg m$^{-2}$ s$^{-1}$, respectively. $P$ is also presented in units of
mm day$^{-1}$ in the paper after a division by the density of liquid water and converted from s$^{-1}$ to
day$^{-1}$. This equation has ignored the change of condensates in the interior of the atmosphere due to condensation and re-evaporation that may be important for precipitation extremes.

Dividing the atmosphere into $N$ layers, the mass-weighted vertical integral of a variable $X$ is denoted as $\{X\} = \sum_{k=1}^{N} X_k \frac{dp_k}{g}$, where $k$ is the index for the atmospheric layer and $dp_k/g$ is the mass per unit area in the $k$th layer. The moisture budget can be rewritten as the sum of $N$ layers from the top of the atmosphere to the surface.

\[
\begin{align*}
P &= -\frac{\partial}{\partial t}\{q\} - \nabla \cdot \{v q\} + E \\
&= -\frac{\partial}{\partial t}\{q\} + \sum_{k} q_k C_k - \{v \cdot \nabla q\} + E
\end{align*}
\]

(3)

where $\{q\}$ is the column water vapor (CWV) in units of kg m$^{-2}$, and $-\{v \cdot \nabla q\}$ depicts horizontal moisture advection. The horizontal mass convergence in the $k$th layer in units of kg m$^{-2}$ s$^{-1}$ is

\[
C_k = -\nabla \cdot (v_k \frac{dp_k}{g})
\]

(4)

It should be noted that we have separated horizontal moisture transport $-\{v \cdot \nabla q\}$ from vertical moisture transport $\sum_k q_k C_k$ due to horizontal mass convergence. Horizontal convergence is directly related to rising air and subsequent condensation above the lifting condensation level (LCL). For a simple advective flow without any frontal or convective lifting there can be a substantial cancellation between horizontal moisture advection $-\{v \cdot \nabla q\}$ and the moisture tendency $\partial \{q\}/\partial t$, simply due to large scale advection.

From Eq. (3), the $e$th percentile of precipitation may be explained from the conditional average of the column moisture budget as

\[
P^e = \sum_{k} q_k^e C_k^e - (\frac{\partial}{\partial t}\{q\} + \{v \cdot \nabla q\})^e + E^e,
\]

(5)

Here the superscript $e$ denotes the conditional average over the precipitation percentile range of $[e_{i-0.5}, e_{i+0.5}]$ in Eq. (1), in which the normalized number of events is $N^e = (e_{i+0.5} - e_{i-0.5})/100$.
and $\sum e N^e = 1$, and the counts of zero precipitation are included in the percentile calculation. As $N^e$ is only a function of the percentile range, it differs from the number of events in evenly spaced precipitation bins. It should be noted that $C_k^e$, which is conditioned onto the $e$th percentile of precipitation, is generally different from the $e$th percentile of $C_k$. Summing over the full probability distribution of precipitation yields the mean precipitation as

$$\sum_e N^e P^e = \sum_e N^e \sum_k q^e_k C_k^e - \sum_e N^e \left( \frac{\partial}{\partial t} \{q\} + \{\mathbf{v} \cdot \nabla q\} \right)^e + \sum_e N^e E^e. \quad (6)$$

### b. Thermodynamic versus dynamic decomposition

In deriving Eq. (5), we have made an assumption $(q_k C_k)^e - q_k^e C_k^e = [(q_k - q_k^e)(C_k - C_k^e)]^e \approx 0$. This is equivalent to assuming that specific humidity and mass convergence for the events falling into the percentile range of $[e_i-0.5, e_i+0.5]$ are uncorrelated. While this is consistent with the statistical decomposition of thermodynamic and dynamic mechanisms using vertical velocities (Emori and Brown 2005; Chen et al. 2011), this is counterintuitive, because $q$ and $C$ are expected to be correlated, i.e., the mass convergence is enhanced by latent heating that increases with specific humidity, and their covariance play an important role in the moisture budget (e.g. Yang et al. 2014b).

We have verified this assumption in the control runs of MPAS CAM4 aquaplanet simulations at three horizontal resolutions. Figure 1 displays the scatter plots of 6 hourly $q_k$ and $C_k$ at the equator and 867 hPa for all the data and then for the data in selected precipitation percentile bins. When all of the data are used, moisture and convergence indeed exhibit the expected positive correlation with the correlation coefficient in the range of [0.31, 0.35] for the three resolutions; higher values of conditionally averaged convergence marked by red circles yield larger conditionally averaged specific humidity. For the same value of specific humidity, the corresponding convergence in the higher resolution runs, which resolve more fine-scale features, is larger. In contrast, when the data
only within a given precipitation percentile bin (e.g., [0, 48.2] in cyan for P28.0 or [99.86, 99.93] in black for P99.9) are plotted, the correlation between convergence and moisture ([0.21, -0.01] for P28.0 and [0.08, 0.18] for P99.9) is much reduced. This gives empirical evidence for ignoring the covariance between \( q^e_k \) and \( C^e_k \) for the \( e \)th percentile in deriving Eq. (5). More physically, this may be understood as the result of the high correlation between precipitation and lower-level convergence and weak correlation between precipitation and specific humidity. In the limit that there is a one-to-one relationship between convergence and precipitation, \( C_k \) is constant in a given precipitation bin, and the correlation between \( q^e_k \) and \( C^e_k \) in this bin is exactly zero. As such, this allows us to clearly separate the moisture (thermodynamic) effect associated with a change in \( q^e_k \) from the circulation (dynamic) effect due to a change in \( C^e_k \). The small covariance term will be further verified in section 4 using the moisture budget for each precipitation percentile.

Unlike vertical moisture transport, there seems to be no simple way of separating the circulation and moisture effects in the horizontal advection of moisture as a function of precipitation intensity. For example, during horizontal mixing of dry and moist air masses at a front, advection involves air masses associated with different precipitation effects. Also, horizontal moisture advection in the absence of front is, at least partly, cancelled by the local CWV tendency, as one would expect from Lagrangian considerations for horizontal advection. Hence, the horizontal advection term in Eqs. (5) and (6) is combined with the local CWV tendency.

Additionally, the conditional moisture budget can be used to examine the precipitation response to climate warming as a function of precipitation percentile. From Eq. (5), the response of the \( e \)th percentile of precipitation to a climate forcing is

\[
\Delta P^e = \sum_k \Delta q^e_k C^e_k + \sum_k q^e_k \Delta C^e_k - \Delta \left( \frac{\partial}{\partial t} \{ q \} + \{ \mathbf{v} \cdot \nabla q \} \right)^e + \Delta E^e, \tag{7}
\]
where $\Delta$ indicates the difference between the perturbed climate and the control climate, when each is conditioned on the $e$th percentile of precipitation in the respective climate. Here we have again ignored the covariance between anomalous moisture $\Delta q^e_k$ and mass convergence $\Delta C^e_k$ for the same reason that moisture and convergence are weakly correlated in a given precipitation bin. The first two terms on the right hand side of Eq. (7) attribute the change in precipitation to the changes in atmospheric moisture and mass convergence, respectively, which we will term as thermodynamic and dynamic contributions, respectively. Summing over the full probability distribution of precipitation yields the mean precipitation change as

$$
\sum_e N^e \Delta P^e = \sum_e N^e \sum_k \Delta q^e_k C^e_k + \sum_e N^e \sum_k q^e_k \Delta C^e_k - \sum_e N^e \Delta \left( \frac{\partial}{\partial t} \{q\} + \{v \cdot \nabla q\}\right)^e + \sum_e N^e \Delta E^e (8)
$$

c. Gross moisture stratification

The theory of gross moist stability (GMS) (e.g. Neelin and Yu 1994; Neelin and Zeng 2000) has shown a couple of vertical modes and associated GMS can explain much of moist dynamics. Thus, the physical interpretation of thermodynamic and dynamic terms is illustrated by separating the magnitude and vertical structure of horizontal mass convergence.

$$C^e_k = C^e \tilde{C}^e_k$$ (9)

Here $C^e_k$ is independent of height. $\tilde{C}^e_k$ is a normalized vertical weighting function for the $e$th percentile of precipitation in the $k$th atmospheric layer, similar to the vertical modes used in defining GMS, but it is diagnosed directly from the conditional mean and hence not necessarily orthogonal. For simplicity, we define the magnitude of mass convergence $C^e_k$ by the standard deviation of $C^e_k$ in the vertical and by the sign of vertical moisture transport.

$$C^e_k = \text{SIGN} \left( \sum_k q^e_k C^e_k \right) \sqrt{\frac{1}{N} \sum_k (C^e_k)^2}$$ (10)

\footnote{The vertical average of mass convergence $C^e_k$ is expected to be small from the mass continuity equation.}
The gross moisture stratification and its change may be obtained as

\[ M^e = \sum_k q^e_k \tilde{C}^e_k, \quad \Delta M^e = \sum_k \Delta q^e_k \tilde{C}^e_k \] (11)

The benefit of using bulk quantities \( C^e_s \) and \( M^e_s \) is illustrated in Fig. 2, in which the vertical structures of \( C^e_k \) and \( q^e_k \tilde{C}^e_k \) at the equator conditioned onto P99.9 are compared between the control run and 3K uniform warming run across three horizontal resolutions. The mass convergence shares a similar baroclinic vertical structure for all the runs examined, justifying the use of a vertical weighting function \( \tilde{C}^e_k \) for the simplification of vertical transport. When \( \tilde{C}^e_k \) is multiplied by \( q^e_k \), the resultant vertical transport term is weighted towards the lower level convergence. Given the little change in vertical structure, \( \tilde{C}^e_k \) and \( q^e_k \tilde{C}^e_k \) are well described by their respective bulk quantities \( C^e_s \) and \( M^e_s C^e_s \) for both their forced changes and sensitivities to horizontal resolutions.

Additionally, the full moisture budget may be simplified to shed light on the mechanisms of precipitation change. If the CWV tendency, horizontal advection, and evaporation can be ignored, as will be shown for precipitation extremes in section 4, Eq. (7) gives an approximation for the fractional change in precipitation extremes as

\[ \frac{\Delta P^e}{P^e} \approx \frac{\Delta M^e_s}{M^e_s} + \frac{\Delta C^e_s}{C^e_s} + \frac{\sum_k q^e_k \Delta \tilde{C}^e_k}{\sum_k q^e_k \tilde{C}^e_k} \] (12)

Assuming that the change in relative humidity is small, the fractional change of gross moisture stratification can be estimated by the CC relationship (i.e., \( \Delta q_{sat}/q_{sat} = \alpha \Delta T \), and \( \alpha \approx 7\% \) for typical tropospheric temperature)

\[ \frac{\Delta M^e_s}{M^e_s} \approx \frac{\sum_k \alpha \Delta T^e_k q^e_k \tilde{C}^e_k}{\sum_k q^e_k \tilde{C}^e_k} \approx \alpha \Delta T^e_s \] (13)

Given that the typical vertical structure of \( \tilde{C}^e_k \) is expected to resemble the first baroclinic mode (Fig. 2), \( \Delta M^e_s/M^e_s \) reflects the change in lower tropospheric moisture and temperature. Here we have assumed the near-surface temperature change \( \Delta T^e_s \) is representative of the lower tropospheric
moisture change. It follows that the thermodynamic term can be approximated as

$$
\sum_k \Delta q_k^e C_k^e = \frac{\Delta M^e}{M^e_*} \sum_k q_k^e C_k^e \approx \alpha \Delta T^e_* M^e_* C^e_* \tag{14}
$$

By contrast, the dynamic term can be decomposed into a change in the magnitude of lower tropospheric mass convergence plus a change in its vertical structure.

$$
\sum_k q_k^e \Delta C_k^e = \left( \frac{\Delta C^e_*}{C^e_*} + \frac{\sum_k q_k^e \Delta \tilde{C}_k^e}{\sum_k q_k^e C_k^e} \right) \sum_k q_k^e C_k^e \approx M^e_* \Delta C^e_* \tag{15}
$$

Here the last approximation has assumed that the dynamic term is dominated by the change in the magnitude of mass divergence rather than its vertical structure. This approximation will be verified in section 5 by comparing the two terms in Eq. (15).

4. The moisture budget for mean and extreme precipitation

The moisture budget is analyzed for mean precipitation and precipitation extremes using the MPAS CAM4 aquaplanet configuration at approximately 240km, 120km and 60km horizontal resolutions. Figure 3a-d shows the climatologies and precipitation extremes of the control simulation and their responses to 3K uniform SST warming. In this idealized warming scenario, the time mean precipitation minus evaporation ($P - E$) response exhibits the well-known thermodynamic mechanism that enhances the wet-dry disparity in the climatology: tropical and midlatitude wet regions become wetter, and subtropical dry regions get drier (e.g. Chou and Neelin 2004; Held and Soden 2006; Chou et al. 2009). Similarly, precipitation extremes in the deep tropics and midlatitude storm tracks, denoted by the 99.9th percentile of precipitation ($P_{99.9}$), increase in the warmer climate, while there is almost no change near $\sim 15^\circ$ latitude where $P_{99.9}$ is minimum in the control runs. These are typical features of the hydrological response to uniform SST warming in aquaplanet models (e.g. Chen et al. 2013). As the model horizontal resolution increases, precipitation shows a sign of dynamical convergence in the extratropics but only qualitative agreement
in the deep tropics, as expected from the critical role of convective parameterizations for tropical precipitation and their dependence on resolution. Interestingly, while most tropical and midlatitude regions with more precipitation extremes in the warmer climate are associated with increased mean $P - E$, some subtropical regions ($\sim 30^\circ$ latitude) are expected to receive enhanced extreme precipitation but less mean $P - E$, indicating a change in the shape of the probability distribution of precipitation in the subtropics, likely due to the change in the probability of non-precipitating events.

The hydrological changes are compared with typical time averaged metrics of atmospheric moisture and near-surface circulation (Fig. 3e-h). The CWV increases are almost proportional to their climatologies, which may be explained by the CC relationship with respect to atmospheric warming (not shown). Meanwhile, the surface meridional wind displays a consistent poleward expansion of tropical Hadley circulations, as indicated by the shift of the zero-crossing latitude between tropical equatorward surface flow and midlatitude poleward flow. Comparing the simulations at three resolutions, CWV converges much faster with resolution than surface meridional wind, because CWV is controlled by temperature and thus less dependent on subgrid parameterizations than circulation.

The role of atmospheric moisture and circulation in the hydrological cycle can be quantified by the moisture budget described in Eqs. (5) and (6). Figure 4 gives the budget for the mean hydrological cycle and precipitation extremes (i.e., P99.9) in the control run and their response to 3K uniform warming at 60km resolution. Summing over the full distribution, $\sum e N^e (P^e - E^e)$ is largely explained by vertical moisture transport $\sum e N^e \sum_k (q_k C_k^e)$, not only for the climatological distribution of tropical and midlatitude wet regions and subtropical dry zones, but also for the wetter-wetter pattern in the warmer climate. Horizontal moisture advection $- \sum e N^e \{v \cdot \nabla q\}^e$ also plays an important role: the equatorward advection of subtropical dry air by the Hadley cell yields
a drying effect throughout the tropics in the control run. This advective drying effect becomes larger in the warmer climate, partly due to increased meridional moisture gradient (inferred from Fig. 3f), and thus leads to an equatorward contraction of the convective margin in the deep tropics through a negative dynamic feedback (e.g. Chou and Neelin 2004; Chou et al. 2009; Su et al. 2017).

Similarly, precipitation extremes are well explained by vertical moisture transport for both the control and perturbed runs (Figs. 4c and 4d). Other factors such as horizontal moisture advection, CWV tendency, and evaporation (not shown) can be ignored except for horizontal moisture advection in high latitudes. Note that while the mean moisture budget is closed very well, there is a small residual in the P99.9 budget. This can be attributed to the interpolation from the model native grids to regular longitude by latitude grids as well as the neglected instantaneous change in condensation and evaporation in the interior of the atmosphere.

The conditional moisture budget is displayed as a function of precipitation percentile in Fig. 5 for both the control run and its response to uniform warming. Two latitudinal bands are selected to compare different changes in the shape of precipitation distribution: mean $P - E$ and P99.9 have same-signed positive trends at the equator (i.e., 0° latitude) but opposite-signed trends in the subtropics (i.e., 30° latitude) (Figs. 4b and 4d). At the equator, $P - E$ is dominated by vertical moisture transport over the entire distribution of precipitation for both the control and perturbed runs, and horizontal advection and CWV tendency can be neglected. However, while the change in vertical moisture transport is responsible for increases in precipitation extremes ($> 99.5$th percentile) in the subtropics in the warmer climate, horizontal advection plays an important role in the decreased precipitation between the 90th and 99th percentiles. This decrease between the 90th and 99th percentiles, weighted by the number of events in each percentile, corresponds to the mean drying trend in Fig. 4b. The CWV tendency acts to reduce the effect of horizontal advection for
precipitation. The combined effects of vertical and horizontal moisture transport in the subtropics
give rise to a decrease in the mean and an increase in the extreme and hence a longer tail in the
probability distribution of precipitation. Interestingly, the change of precipitation displays a rapid
increase with higher percentiles irrespective of the latitude considered. This can be attributed to
the thermodynamic effect of increasing atmospheric moisture and will be discussed in detail with
respect to Fig. 11.

In summary, the moisture budget analysis shows the leading role of vertical moisture transport
for mean $P - E$ and precipitation extremes, while horizontal moisture advection is important for
mean precipitation or the shape of precipitation distribution especially in the subtropics.

5. Thermodynamic and dynamic mechanisms for the full distribution of precipitation

a. Precipitation extremes

Given the dominant balance between precipitation extremes and vertical moisture transport, we
first examine individual effects of moisture and mass convergence on precipitation extremes. Fig-
ure 6 displays the latitude-pressure cross section of temperature change in response to 3K SST
warming and the associated change in vertical moisture transport due to increased moisture. The
resemblance between conditionally averaged temperature onto $P_{99.9}$ and time mean temperature
suggests that the thermodynamic effect exerts similar influences on extreme and mean precipita-
tion. The temperature response conditioned onto $P_{99.9}$ displays a familiar pattern of tropospheric
warming and stratospheric cooling, and the subtropical warming in the stratosphere is likely caused
by a change in residual circulation driven by the change in eddy forcing (Yang et al. 2014a). Tro-
pospheric warming leads to an increase in atmospheric moisture that is concentrated in the lower
troposphere, which, in conjunction with the baroclinic structure of mass convergence for $P_{99.9}$
(Fig. 2), causes a net wetting trend in P99.9 after the vertical moisture transport is integrated vertically (Fig. 6c).

In contrast, the change in mass convergence associated with P99.9 exhibits an upward and poleward shift in its baroclinic structure (Fig. 7). Coupled with the vertical profile of specific humidity, the change in mass convergence leads to a net drying trend in P99.9 in the subtropics after the vertical moisture transport is vertically integrated. In comparison with the thermodynamic effect, the dynamic change is smaller in magnitude (cf. Figs. 6c and 7b). The change of the dynamic term can be further separated as a change in the magnitude of convergence independent of height and a change in its vertical structure. Although the spatial pattern of the latter is more similar to the total dynamic change, the former contributes largely to the subtropical drying trend in P99.9 (Fig. 7c), and the latter has a smaller effect on P99.9 due to the cancellation of anomalies of different signs in the vertical (Fig. 7d). This point will be further clarified in the discussion of Figs. 8e and 8f.

Combining the thermodynamic and dynamic components, the fractional change in precipitation extremes (Fig. 8a) can be explained by vertical moisture transport (Fig. 8b) via Eq. (12) across different horizontal resolutions, except for high latitudes where horizontal moisture transport is larger than vertical transport (see Fig. 4d). The total fractional change in P99.9 (Fig. 8a) deviates considerably from a flat thermodynamic increase expected from the CC relationship (≈7% per K warming) and 3K uniform warming, but its thermodynamic component, the fractional change in gross moisture stratification, agrees well with the CC expectation (Fig. 8c). This also helps confirm that the choice of breaking the budget out by percentiles of precipitation is reasonably reflecting the underlying dynamics when applied term by term. Interestingly, the fractional change in CWV (≈10% per K warming) is greater than the CC expectation, because gross moisture stratification is weighted toward the lower troposphere but CWV includes the entire troposphere with a larger fractional change in the upper troposphere due to elevated warming. The dynamic change, in
contrast, displays a drying trend at $\sim 15^\circ$ and a wetting trend at $\sim 55^\circ$ (Fig. 8d). This may be interpreted by the poleward expansion of the Hadley cell or the poleward shift in midlatitude storm tracks (Fig. 3h). Moreover, the dynamic change can be mostly explained by the change in the magnitude of mass convergence (Fig. 8e), with only smaller contributions by the change in its vertical structure (Fig. 8f). The exception is the large change at $\sim 15^\circ$ due to the vertical structure change of convergence in 120km and 60km runs, but this large fractional change can be partly attributed to the small denominator, P99.9 minimum at $\sim 15^\circ$ in the control run, and thus it has a small impact on the absolute value of P99.9. Overall, the dynamic change is approximately cancelled by the thermodynamic change near $\sim 15^\circ$, leaving only a small fractional change in P99.9, whereas the two effects work constructively in midlatitudes to yield super-CC fractional increases in P99.9.

In conclusion, the fractional change in precipitation extremes can be decomposed into the changes in thermodynamic and dynamic components, respectively, in which the thermodynamic component can be predicted from the CC relationship and near-surface temperature change, and the dynamic change is controlled by the magnitude of lower level mass convergence.

b. Full probability distribution of precipitation

The thermodynamic and dynamic analysis for precipitation extremes above is readily extended to the full probability distribution of precipitation (Fig. 9). From low to high precipitation percentiles, precipitation exhibits two robust centers in the tropics and midlatitudes, and the midlatitude center is tilted equatorward with increasing precipitation percentiles. Under uniform SST warming, precipitation (Fig 9a) is intensified everywhere except for the drying trend maximizing between 90th and 99th percentiles on the equatorward flank of the midlatitude precipitation band, consistent with Fig. 5d. The change in conditionally averaged evaporation is important only for
low precipitation percentiles, where $P^e - E^e$ is weakly positive or negative (Fig 9b). As such, the net change of $P - E$ over the full precipitation distribution is, to leading order, explained by vertical moisture transport (cf. Figs 9a and 9c), while the combination of horizontal moisture advection and CWV tendency is secondary (Fig. 9d). However, horizontal advection gives a drying trend in the subtropics for both the control run and the forced response to uniform warming and it does not vary much with precipitation percentile, and thus it can contribute greatly to mean precipitation change. Physically speaking, this reflects the nature of warm/moist core precipitation, where the mass convergence is associated with a lower-level cyclonic circulation that advects dry air into the moist center.

The vertical moisture transport is explicated as follows. On the one hand, the thermodynamic change to uniform SST warming amplifies the pattern of vertical transport in the control run, as indicated by the same pattern for the change (shading) and the control (black lines) in Fig. 9e. For a given increase in moisture abundance the heaviest precipitation of the control climate will experience the largest increase in a warmer climate, which may be thought of as a generalized 'wet-get-wetter' mechanism. This thermodynamic change is well predicted by the change in conditionally averaged temperature via the CC relationship (cf. Figs. 9e and 9g). On the other hand, the dynamic change exhibits a drying trend on the equatorward flank of midlatitude storm tracks (Fig. 9f), which corresponds to the reduced amplitude of lower level convergence there (Fig. 9h), and possibly the poleward expansion of the subtropical dry zone, indicated by the surface mean meridional circulation change (Fig. 3h).

To better compare the thermodynamic and dynamic responses to uniform SST warming, they are illustrated as a function of latitude for mean and extreme precipitation (Fig. 10) and as a function of precipitation percentile at the equator (i.e., $0^\circ$ latitude) and in the subtropics ($\sim 30^\circ$ latitude) (Fig. 11). In terms of mean precipitation change, the dynamic change is important for
the poleward expansion of the subtropical dry zone that cannot be explained by the wet-get-wet mechanism but by the circulation shift. While the change in precipitation extremes is mostly explained by the thermodynamic component due to atmospheric warming, the dynamic component is needed to explain reduced precipitation extremes on the subtropical jet’s equatorward flank and increased extremes on the jet’s poleward flank. Near the equator, the change in circulation contribution slightly tightens the equatorial increase in P99.9. As far as the precipitation distribution is concerned (Fig. 11), the change in $P - E$ at the equator is largely explained by the thermodynamic change due to conditionally averaged temperature through the CC relationship, and the change in circulation contributes only modestly, even at the highest percentiles. The thermodynamic change in 30° latitude, however, is substantially offset by the dynamic change that introduces a drying tendency above the 90th percentile. The combination of the dynamic term and horizontal moisture advection (Fig. 5d) produces a large drying in the subtropics against the positive thermodynamic contribution to more extreme precipitation, resulting in decreased mean $P - E$ but increased precipitation extremes.

Figures 10 and 11 show that the thermodynamic and dynamic components are predicted by their approximations due to the temperature change in Eq. (14) and the lower level divergence change in Eq. (15), respectively. If we assume that the entire atmospheric column is saturated during extreme precipitation, precipitation extremes can be approximated as

$$P^e \approx \sum_k q_{sat}(T_k^e)C_k^e = M_{sat}(T_k^e)C_*^e$$  \hspace{1cm} (16)$$

where $q_{sat}(T_k^e)$ is the saturation specific humidity that is determined by conditionally averaged temperature, and $M_{sat}(T_k^e) = \sum_k q_{sat}(T_k^e)\tilde{C}_k^e$ is the saturation gross moisture stratification computed from the saturation specific humidity and baroclinic vertical structure of mass convergence. From the scatter plot of P99.9 versus $M_{sat}(T_k^e)C_*^e$ in Fig. 12, we see that the prediction for precipitation
extremes works as a reasonable first approximation for both the control run and the response to uniform SST warming across different horizontal resolutions. It is worth noting that the prediction tends to underestimate P99.9 likely due to ignoring other terms in the moisture budget (Figs. 4c and 4d), but this bias can be reduced by a regression coefficient of 1.14 to account for the neglected terms.

This scaling is qualitatively similar to the scaling proposed by O’Gorman and Schneider (2009), but the proposed scaling above is based on conditionally averaged Eulerian statistics in contrast to the assumed balance in the updraft along a moist adiabat in O’Gorman and Schneider (2009). This can lead to different thermodynamic interpretations focusing on the lower tropospheric moisture that varies with the CC relationship in this paper versus the vertical gradient of lower-level saturation water vapor that is controlled by the moist adiabatic lapse rate (O’Gorman and Schneider 2009).

### 6. Conclusions

In this paper we have developed a conditional mean column-integrated moisture budget of the atmosphere for the full probability distribution of precipitation. This new formulation extends the traditional mean precipitation budget (e.g. Seager et al. 2014) to the budget of precipitation of a given percentile in terms of conditionally averaged vertical moisture transport, horizontal moisture advection, CWV tendency, and evaporation. Similar to the GMS theory for mean precipitation (e.g. Neelin and Yu 1994; Neelin and Zeng 2000), the conditional vertical moisture transport is proportional to the gross moisture stratification that is defined by the moisture profile projected onto the vertical structure of conditionally averaged horizontal mass convergence. Analysis is performed on idealized aquaplanet model simulations under 3K uniform warming across different horizontal resolutions. Saturation gross moisture stratification multiplied by mass convergence can...
predict precipitation extremes to a reasonable degree of approximation for all of our simulations (Fig. 12).

This formulation gives a consistent interpretation of thermodynamic and dynamic mechanisms for both mean precipitation and precipitation extremes. The conditionally averaged specific humidity and horizontal mass convergence onto precipitation intensity, as a result of weakly correlated moisture and convergence in a given precipitation bin (bottom panels of Fig. 1), yield a clear separation between the moisture (thermodynamic) and circulation (dynamic) effects of vertical moisture transport. The thermodynamic response to idealized climate warming can be understood as a generalized 'wet-get-wetter' mechanism that the heaviest precipitation of the probability distribution is enhanced most from increased gross moisture stratification, at a rate controlled by the change in lower tropospheric moisture rather than column moisture. In the deep tropics, the change in $P - E$ over the full precipitation distribution is largely explained by the thermodynamic change from conditionally averaged temperature by the CC relationship.

The dynamic effect, in contrast, can be interpreted by shifts in large-scale atmospheric circulations such as the Hadley Cell circulation or midlatitude storm tracks. This effect is dominated by the change in the magnitude of mass convergence rather than its vertical structure. The horizontal moisture advection, albeit of secondary role, is important for regional precipitation especially for the change in mean precipitation. Furthermore, horizontal advection of dry air may suppress the convection at the tropical convective margin that gives a negative dynamic feedback on precipitation (e.g. Chou and Neelin 2004; Chou et al. 2009). In the subtropics, the dynamic term in conjunction with horizontal moisture advection produces a large drying effect against the positive thermodynamic contribution to more extreme precipitation, resulting in decreased mean $P - E$ but increased precipitation extremes.
While the thermodynamic component of precipitation change is well understood by the change in temperature (O’Gorman and Schneider 2009; Pfahl et al. 2017), the moisture budget alone does not offer much insight on the circulation change. For example, more realistic CMIP5 models (Pfahl et al. 2017) and CESM LENS simulations (Norris et al. 2018) predict a large strengthening of ascent in the deep tropics in a warming climate, while the equatorial circulation response to uniform SST warming in aquaplanet simulations is small (Fig. 8d) in spite of consistent changes in mass convergence and vertical transport (Fig. 7). Because the moist static energy budget can be used to constrain the regional circulation change and mean precipitation (e.g. Chou and Neelin 2004; Chou et al. 2009), a similar conditional energy budget analysis needs to be carried out, which may shed light on the energetic constraint of convection and precipitation extremes in response to climate warming.
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References


LIST OF FIGURES

Fig. 1. Scatter plots for 6 hourly specific humidity, $q_k$, versus horizontal mass convergence, $C_k$, at 0° latitude and 867 hPa simulated in the control runs of the MPAS CAM4 aquaplanet model at three horizontal resolutions: (left) 240km, (middle) 120km, and (right) 60km. The top panels include all the data, and red circles mark the conditional means for the 11 precipitation percentile bins used in this study. The bottom panels consist of data only in the two percentile bins of (cyan) [0.48,2] and (black) [99.86,99.93], which are set by Eq. (1) and denoted by their approximate central percentile values as P28.0 and P99.9, respectively. While $q_k$ and $C_k$ are correlated for all the data as a whole, the correlation is much reduced for the subset of data in a given precipitation bin. The correlation coefficients are color coded and given after 'r ='.

Fig. 2. The vertical structure of (top) horizontal mass convergence, $C_k^e$, and (bottom) vertical moisture transport, $q_k^e C_k^e$, conditioned onto the 99.9th percentile of precipitation (P99.9), for the (blue lines) control run and (red lines) 3K warming run at 0° latitude for three horizontal resolutions: (left) 240km, (middle) 120km, and (right) 60km. The vertical dashed lines indicate the amplitude of divergence $C_k^e$ (i.e., the standard deviation of the vertical variation in $C_k^e$) in the top panels and the amplitude of total vertical transport $M_k^e C_k^e$ (scaled by the factor of 1/5 for comparison) in the bottom panels.

Fig. 3. (Left) control simulations and (right) anomalies relative to the control for the responses to 3K uniform SST warming at three horizontal resolutions (i.e., 240km, 120km, and 60km): (a)(b) zonally averaged precipitation minus evaporation (P-E, mm day$^{-1}$), (c)(d) 99.9th percentile of precipitation (P99.9, mm day$^{-1}$), (e)(f) column-integrated water vapor (CWV, mm), and (g)(h) surface meridional wind (V, m s$^{-1}$).

Fig. 4. The moisture budget for (left) control simulations and (right) anomalies relative to the control in response to 3K uniform SST warming at 60km resolution: (a)(b) the mean hydrological cycle and (c)(d) precipitation extremes at the 99.9th percentile (P99.9). Individual terms of the moisture budget are described in Eqs. (6) and (8) for the mean budget and in Eqs. (5) and (7) for precipitation extremes. The leading order terms of the moisture budget are precipitation minus evaporation $P - E$ and vertical moisture transport $\Sigma_k(q_k^e C_k^e)$. The time-mean budget terms in the top panels are weighted by the number of events $N^*$ in each percentile range. The contribution of $E^*$ to the P99.9 budget in the bottom panels is negligible.

Fig. 5. The moisture budget for (left) control simulations and (right) anomalies relative to the control in response to 3K uniform SST warming as in Fig. 4, but as a function of precipitation percentile at (a)(b) 0° and (c)(d) 30° latitude.

Fig. 6. The thermodynamic response to 3K warming at 60km resolution: (a) time averaged temperature (K), (b) conditionally averaged temperature (K) onto P99.9, and (c) conditionally averaged thermodynamic term (kg m$^{-2}$ s$^{-1}$). The control simulation is shown in black lines, and the contour values in (c) are $-1,1,3,10 \times 10^{-4}$ kg m$^{-2}$ s$^{-1}$.

Fig. 7. The dynamic response to 3K warming conditioned onto P99.9 at 60km resolution: (a) horizontal mass convergence, (b) dynamic term, (c) dynamic term due to the change in the magnitude of convergence, and (d) dynamic term due to the change in the vertical structure of convergence. Conditionally averaged mass convergence is $C_k^e = C_k^e C_k^e$, where $C_k^e$ denotes the magnitude and $\tilde{C}_k^e$ denotes the vertical structure of mass convergence. The control simulation is shown in black lines, and the contour interval (CI) is $2 \times 10^{-2}$ kg m$^{-2}$ s$^{-1}$ in (a) and contour values in (c) are $-1,1,3,10 \times 10^{-4}$ kg m$^{-2}$ s$^{-1}$ in (b)-(d).
Fig. 8. The fractional change of precipitation extremes, P99.9, and the approximate moisture budget, $P^e \approx \sum_k (q_k C_k^*)$, in response to 3K warming at three horizontal resolutions (i.e., 240km, 120km, and 60km): (a) $\Delta P^e / P^e$, (b) the sum of the changes in gross moisture stratification $\Delta M^*_e / M^*_e = \sum_k \Delta q_k C_k^* / \sum_k q_k C_k^*$ and mass convergence $\sum_k q_k \Delta C_k^* / \sum_k q_k C_k^*$, (c) gross moisture stratification $\Delta M^*_e / M^*_e$ and CWV, (d) mass convergence $\sum_k q_k \Delta C_k^* / \sum_k q_k C_k^*$, (e) the magnitude of convergence $\Delta C^*_e / C^*_e$, and (f) the vertical structure of convergence $\sum_k q_k \Delta C_k^* / \sum_k q_k C_k^*$. Note (d)=(e)+(f).

Fig. 9. The moisture budget as a function of precipitation percentile and latitude at 60km resolution: (a) precipitation (P), (b) precipitation minus evaporation (P-E), (c) vertical moisture transport, (d) horizontal moisture advection minus the CWV tendency, (e) thermodynamic term, (f) dynamic term, (g) thermodynamic prediction from the CC relation, and (h) the magnitude of lower tropospheric mass convergence. Note (c)=(e)+(f). The control simulation is shown in black lines, and the contour values are $-3, 0, 3, 10, 30, 100, 300$ mm day$^{-1}$ in (a)-(g) and $-1.2, 0, 1.2, 4, 12 \times 10^{-3}$ kg m$^{-2}$ s$^{-1}$ in (h).

Fig. 10. The thermodynamic and dynamic responses of (a) mean precipitation and (b) precipitation extremes to 3K warming at 60km resolution. The thermodynamic term is shown in red and dynamic term in blue, and the solid lines are direct calculations and dotted lines are approximations in Eqs. (14) and (15).

Fig. 11. As in Fig. 10, but for the thermodynamic and dynamic responses as a function of precipitation percentile at (a) 0$^\circ$ and (b) 30$^\circ$ latitude.

Fig. 12. Scatter plot of precipitation extremes, P99.9, versus vertical moisture transport, $M^*_e (T_k^*) C_k^*$, at three horizontal resolutions (i.e., 240km, 120km, and 60km) for both control simulations and under 3K uniform SST warming. For each run, a data point represents the P99.9 value and corresponding vertical moisture transport at one latitude. $M^*_e (T_k^*)$ is the saturation gross moisture stratification from conditionally averaged temperature. The black line gives the best least squares fit with a regression coefficient of 1.14.
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